STOCKHOLMS UNIVERSITET
FYSIKUM

Examination in Condensed Matter Physics I, FK3004, 7.5 hp (exam 4.5 hp, lab 3 hp)
Thursday, June 5, 2008, 09.00-15.00.

Allowed help:
- periodic table and fundamental constants (distributed)
- formula sheet (distributed)
- pocket calculator, BETA / mathematics handbook or similar

Instructions:
All solutions should be easy to read and have enough details to be followed. The use of nontrivial formulas from the formula sheet should be explained. *Summarize each problem* before its solution, so that the solution becomes self-explained. State any assumptions and any eventual interpretation of a problem formulation.

Good luck! / A.R.

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1. The sodium chloride (NaCl) structure can be described using an *fcc* lattice with a basis of two ions at $\mathbf{0}$ and $(a/2)(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$, respectively. Assume that the ions can be regarded as impenetrable spheres with definite radii r_{Na} and r_{Cl}.
a) Make a simple sketch of the ion arrangement in the (100) plane. Express the nearest neighbor distance as a function of lattice parameter a. (1p)
b) Given $r_{\mathrm{Na}} = 0.95$ Å and $r_{\mathrm{Cl}} = 1.81$ Å, verify that the Cl ions make contact only with the Na ions and calculate the lattice parameter for NaCl. (1.5p)
c) A simple cubic structure has a filling fraction of 52 %. Find the corresponding filling fraction for NaCl. (1.5p)

2. a) Show that the Fermi energy is given by

$$\varepsilon_F = \frac{\hbar^2 (3\pi^2 n)^{2/3}}{2m}$$

for a metal with n valence electrons per volume. (1.5p)
b) Copper (Cu) has a Fermi temperature of 8.16×10^4 K. Determine the lattice parameter of Cu using this information and suitable assumptions. (1.5p)
c) The electronic specific heat is linear in temperature, $c_{v,\mathrm{el}} = \gamma T$. Use the Lorenz number $L = \pi^2 k_B^2 / 3e^2$ and Wiedemann-Franz law to express γ in terms of ε_F. Describe used relations. (1p)

3. a) Show that the volume v_g of the reciprocal lattice primitive cell is $v_g = (2\pi)^3 / v_c$, where v_c is the volume of the direct lattice primitive cell. (1.5p)
b) In an x-ray diffraction experiment on iron powder, the first observed diffraction peak was located at $2\theta = 44.60°$. Determine the maximum number of diffraction peaks that could possibly be observed in this experiment and give their expected locations. (2.0p)
c) If the wavelength λ could be increased, at what λ would it no longer be possible to see any diffraction peaks? (0.5p)

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4. The internal energy for a monoatomic, one-dimensional crystal with N atoms and longitudinal interactions between nearest neighbors can be written as

$$U = \int_0^{\omega_{\max}} \frac{\hbar\omega}{\exp\left(\hbar\omega/k_{\mathrm{B}}T\right) - 1} \cdot \frac{2N}{\pi\sqrt{\omega_{\max}^2 - \omega^2}} d\omega$$

a) Make suitable assumptions and show this. (2.5p)
b) Describe the basic assumptions of the Debye model and find an expression for the Debye frequency ω_D for a one-dimensional system. (1.5p)

5. a) The paramagnetic susceptibility χ of rare-earth ions at high temperature is proportional to the square of the effective Bohr magneton number p and inversely proportional to temperature. The ions Ce^{3+}, Gd^{3+}, and Dy^{3+} have the electron configurations $4f^1 5s^2 p^6$, $4f^7 5s^2 p^6$, and $4f^9 5s^2 p^6$. One of these ions has a measured Bohr magneton number close to 8.0. Which one? (1.5p)
b) Derive an expression for the Curie temperature in the mean field approximation. Assume an exchange field $B_E = \mu_0 \lambda M$. Describe your starting point. (1.5p)
c) Discuss how magnetic ions interact. (1p)

6. a) Superconductors can be divided into two groups, type-I and type-II, depending on their behavior in magnetic fields. Describe the H-T phase diagram of the two groups. Also discuss what vortices are and why they appear. (1.0p)
b) Assume a simple cubic, divalent metal. Illustrate how the Fermi surface should change with increasing ion lattice potential by sketching (in 2D) the Fermi surface for the three cases of free electrons, a weak potential, and a thought semiconductor. Indicate the 1st and 2nd Brillouin zones, hole pockets, and electron pockets. (1.5p)
c) Discuss what measurements of the Hall effect, electrical resistivity, and optical absorption can tell about a semiconductor. (1.5p)