STOCKHOLMS UNIVERSITET
FYSIKUM

Examination in Condensed Matter Physics FY3600, part I, 5p (exam 3p, lab 2p)
Wednesday, June 20, 2007, 09.00-15.00.

Allowed help:
- periodic table and fundamental constants (distributed)
- formula sheet (distributed)
- pocket calculator, BETA / mathematics handbook

Instructions:
All solutions should be easy to read and have enough details to be followed. The use of nontrivial formulas from the formula sheet should be explained. *Summarize each problem* before its solution, so that the solution becomes self-explained.

Good luck! / A.R.

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1. The atoms in a lattice can be modelled as hard spheres.
a) Calculate the filling fraction of such atoms arranged in *bcc* and *diamond* structure, respectively. (2p)
b) What are the coordination numbers for the atoms in these structures? (0.5p)
c) The *hcp* structure is close-packed. Should this correspond to a lower or higher coordination number? Motivate! (0.5p)
d) The diamond structure does not itself correspond to a Bravais lattice, but can be described as a cubic Bravais lattice with a basis of 8 atoms. However, another Bravais lattice exists that could be used together with a smaller cell / basis to generate the diamond structure. Find the cell volume for this smallest possible cell expressed in the conventional (cubic) lattice parameter a. Motivate clearly. (1p)

2. a) Find expressions for the density of states $g(\varepsilon)$ for electrons in one, two, and three dimensions. Verify that $g(\varepsilon)$ becomes constant in the two-dimensional case. Use the free electron model. (1.5p)
b) Find corresponding expressions for the Fermi energy ε_F in 2D and 3D. (1p)
c) Show the following reciprocal lattice property:

$$d(hkl) = \frac{2\pi}{|\mathbf{G}(hkl)|}$$

where d is the distance between real space lattice planes with Miller indices (hkl) and $\mathbf{G}$ is a reciprocal lattice vector. (1.5p)

3. a) Describe in general terms the difference between Fermi energy and chemical potential μ. (0.5p)
b) Use Sommerfeld expansion technique to show that $\mu - \varepsilon_F \propto T^2$ in the free electron model. Determine the prefactor. Answer in such a way that the prefactor contains ε_F. (2.5p)
c) Describe the concept of thermopower and its typical temperature dependence. (1p)

4. a) Show that the thermal expansion of a lattice with harmonic interactions is zero. State your assumptions. (2p)
b) Describe the difference between sound propagation and heat conductivity in electrical isolators. (2p)

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5. a) Describe shortly the tightbinding model. When should it be used? What main band feature is the model describing? (1p)
b) Answer the same questions as in **a**), but for the nearly free electron model. (1p)
c) If the central equation close to a Brillouin zone boundary is solved, one obtains the following expression for $\varepsilon(\mathbf{k})$:

$$\varepsilon = \frac{1}{2}\left(\varepsilon^0_{\mathbf{k}-\mathbf{G}} + \varepsilon^0_{\mathbf{k}}\right) \pm \sqrt{\frac{1}{4}\left(\varepsilon^0_{\mathbf{k}-\mathbf{G}} - \varepsilon^0_{\mathbf{k}}\right)^2 + U^2}.$$

Explain the used notations ($\varepsilon^0_{\mathbf{k}}$, U, $\mathbf{G}$) and show that the group velocity vanishes when $\mathbf{k} \to \mathbf{G}/2$. (1p)
d) For small wave vectors, a certain band of a one-dimensional crystal is described by

$$\varepsilon(k) = \frac{\hbar^2}{ma^2}\left(\frac{7}{8} - \cos ka + \frac{1}{8}\cos 2ka\right) \quad ,$$

where a is the lattice parameter. Determine the effective mass at the bottom of the band. (1p)

6. a) Use Hund's rules to write the spectroscopic notation ($^{2S+1}L_J$) for the ground states of the ions Cu^+ and Eu^{3+}. Motivate clearly! (1.5p)
b) Describe the types of magnetism of independent magnetic moments (h-k) by placing them together with the words (a-g) in a tree structure: (a) metals, (b) insulators, (c) filled shells, (d) partially filled shells, (e) $J = 0$, (f) $J \neq 0$, (g) electron spins, (h) Langevin diamagnetism, (i) Pauli paramagnetism, (j) free spin paramagnetism, (k) van Vleck paramagnetism. (1p)
c) Describe shortly the following concepts of superconductivity: superconducting gap, Cooper pairs, tunneling, isotope effect, condensation energy, vortices. (1.5p)