

**Examination in Condensed Matter Physics FY3600, part I (5p)**

Monday, March 26, 2007 (Example exam)

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**Allowed help:**

- periodic table and fundamental constants (distributed)
- formula sheet (distributed)
- pocket calculator, BETA / mathematics handbook

**Instructions:**

All solutions should be easy to read and have enough detail to be followed. The use of nontrivial formulas from the formula sheet should be explained. Summarize each problem before its solution, so that the solution becomes self-explained.

Good luck! / A.R.

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**1.** A certain monoatomic metal with *fcc* structure has a lattice parameter  $3.267 \text{ \AA}$  at  $730^\circ\text{C}$ . When the temperature is increased to  $1067^\circ\text{C}$ , the structure has changed to *bcc* with a lattice parameter  $2.784 \text{ \AA}$ .

**a)** What is the relative change in density when going from  $730^\circ\text{C}$  to  $1067^\circ\text{C}$ ? (2.5p)

**b)** Calculate the nearest neighbor distances for the two structures. (1.5p)

**2.** A crystal with simple cubic (*sc*) structure has a lattice parameter  $a = 3.7 \text{ \AA}$ . Assume the free electron model.

**a)** Calculate the bandwidth for the lowest energy band. Give the answer in eV (1.5p)

**b)** Which is the highest number of valence electrons per atom for which the second-lowest energy band remains empty? (1.5p)

**c)** What is the Fermi energy for this number of valence electrons per atom? (1p)

**3.** A polycrystalline sample with bodycentered tetragonal structure was studied with monochromatic x-ray,  $\lambda = 1.5405 \text{ \AA}$ . The four lowest Bragg angles were measured to  $\theta = 21.00^\circ$ ,  $22.06^\circ$ ,  $28.78^\circ$ , and  $32.09^\circ$ .

**a)** Give an expression for a general reciprocal lattice vector  $\mathbf{G}(hkl)$  for the tetragonal lattice, which has lattice vectors  $a\hat{x}$ ,  $a\hat{y}$ , and  $c\hat{z}$ . (0.5p)

**b)** Start with the diffraction condition  $\Delta\mathbf{k} = \mathbf{G}$  and deduce the quadratic form for a tetragonal lattice. (1.5p)

**c)** For *bcc* structures, the allowed reflexes have  $h + k + l = 2n$ , where  $n$  is an integer. Motivate that this is also the case for the bodycentered tetragonal structure. (0.5p)

**d)** Index the structure and determine the lattice parameters  $a$  and  $c$ . (1.5p)

**4. a)** Explain shortly the following concepts: Phonon, dispersion relation, optical branch, harmonicity, zero-point vibrations (2.5p)

**b)** Show how to obtain the Dulong-Petit value of the high-temperature specific heat. Describe your starting point. (1.5p)

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**5.** An intrinsic, direct gap semiconductor has  $\varepsilon_G = 0.8$  eV,  $m_e = 0.3m$ , and  $m_h = 0.6m$ .

**a)** Make a sketch of the bands (energy vs.  $k$ ) that reflect the given effective masses. Indicate band gap, valence band, conduction band, and chemical potential. (1.5p)

**b)** Deduce an expression for the temperature dependence of the chemical potential. Motivate your starting point. (2.5p)

**6. a)** Show that the magnetic susceptibility of free conduction electrons is proportional to the density of states at the Fermi energy. (2.5p)

**b)** Describe the isotope effect for superconductors and explain why an experiment to determine the isotope effect is important for understanding superconductivity (1.5p)