What is topological matter & why do we care?

Part 2: fractionalization & applications?

Eddy Ardonne

WfSW:QT
2014-08-29

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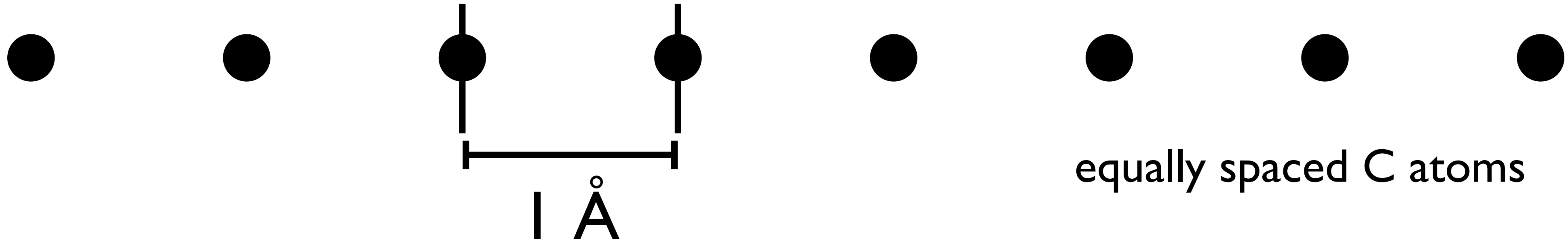
Famous example: polyacetylene (Su, Schrieffer, Heeger, 1979)

Polyacetylene

Polyacetylene consists of a chain of carbon atoms, with alternating single and double bonds, long and short.



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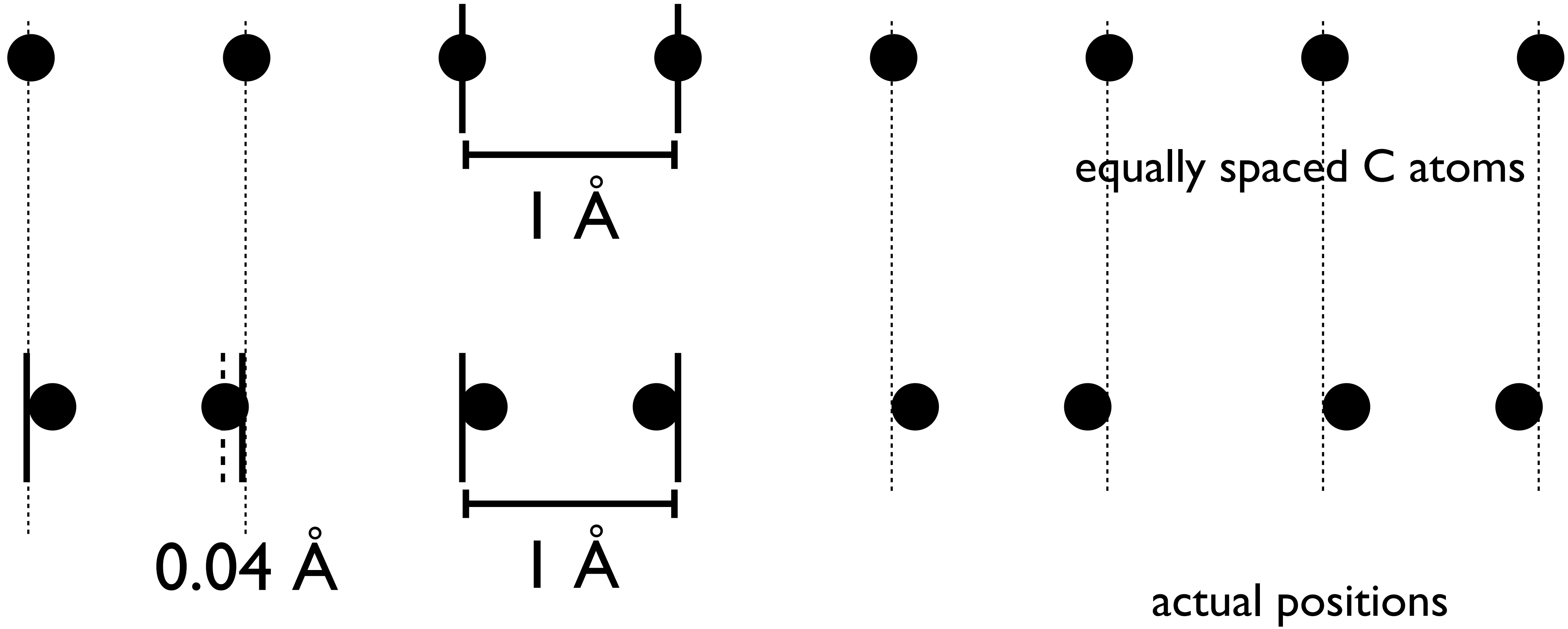

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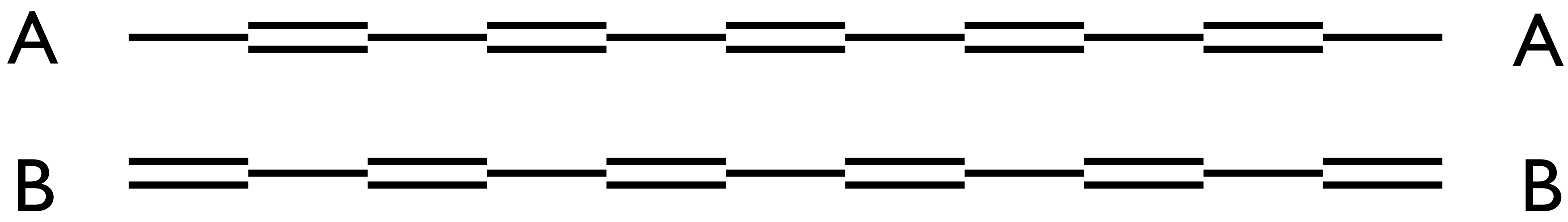

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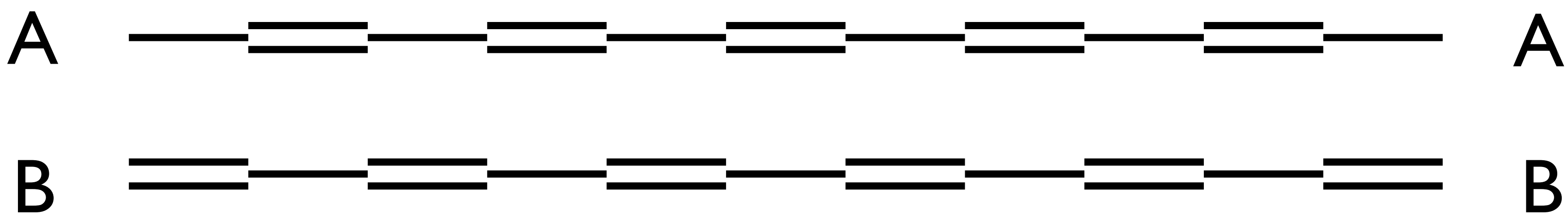

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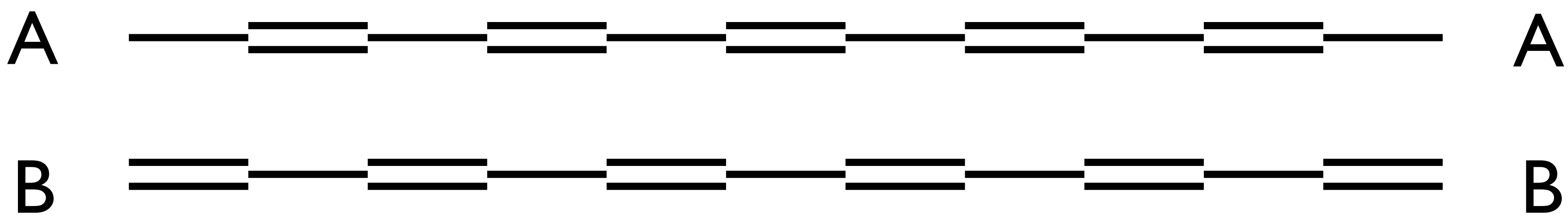

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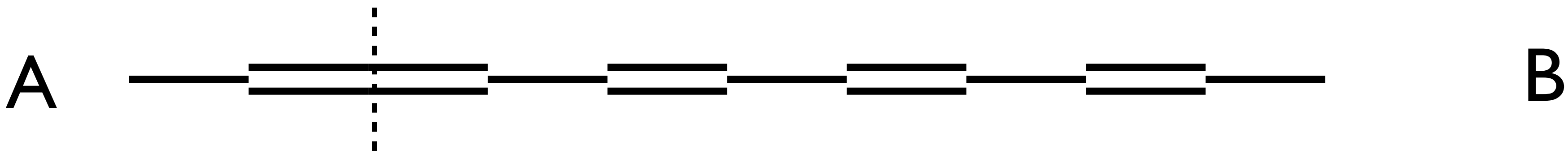

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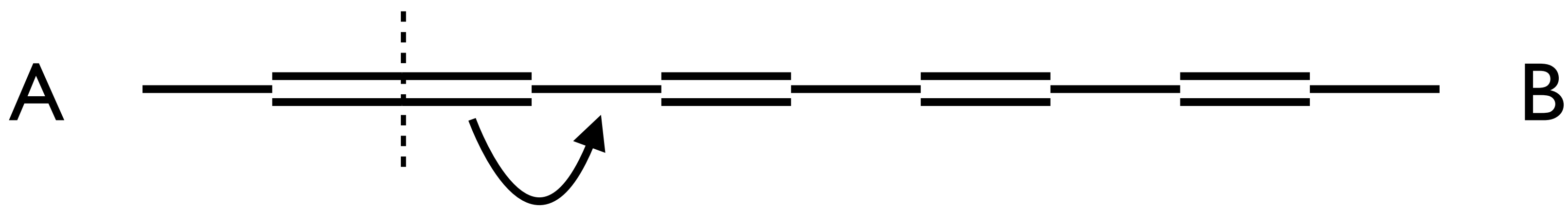

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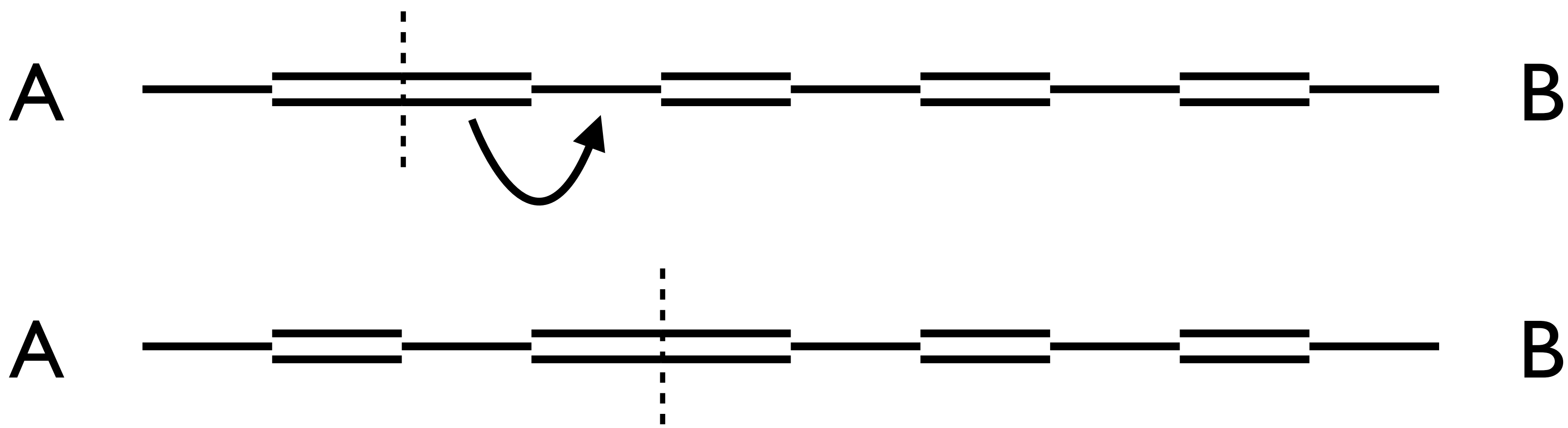

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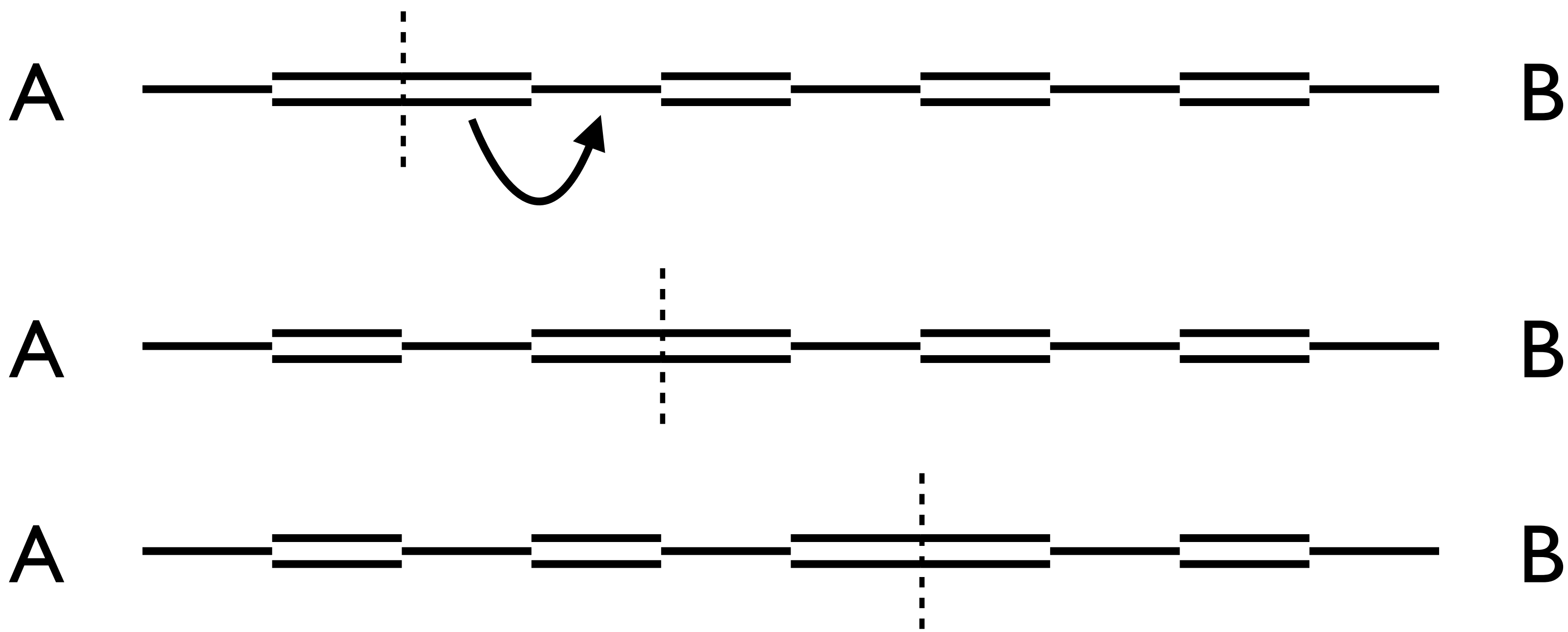

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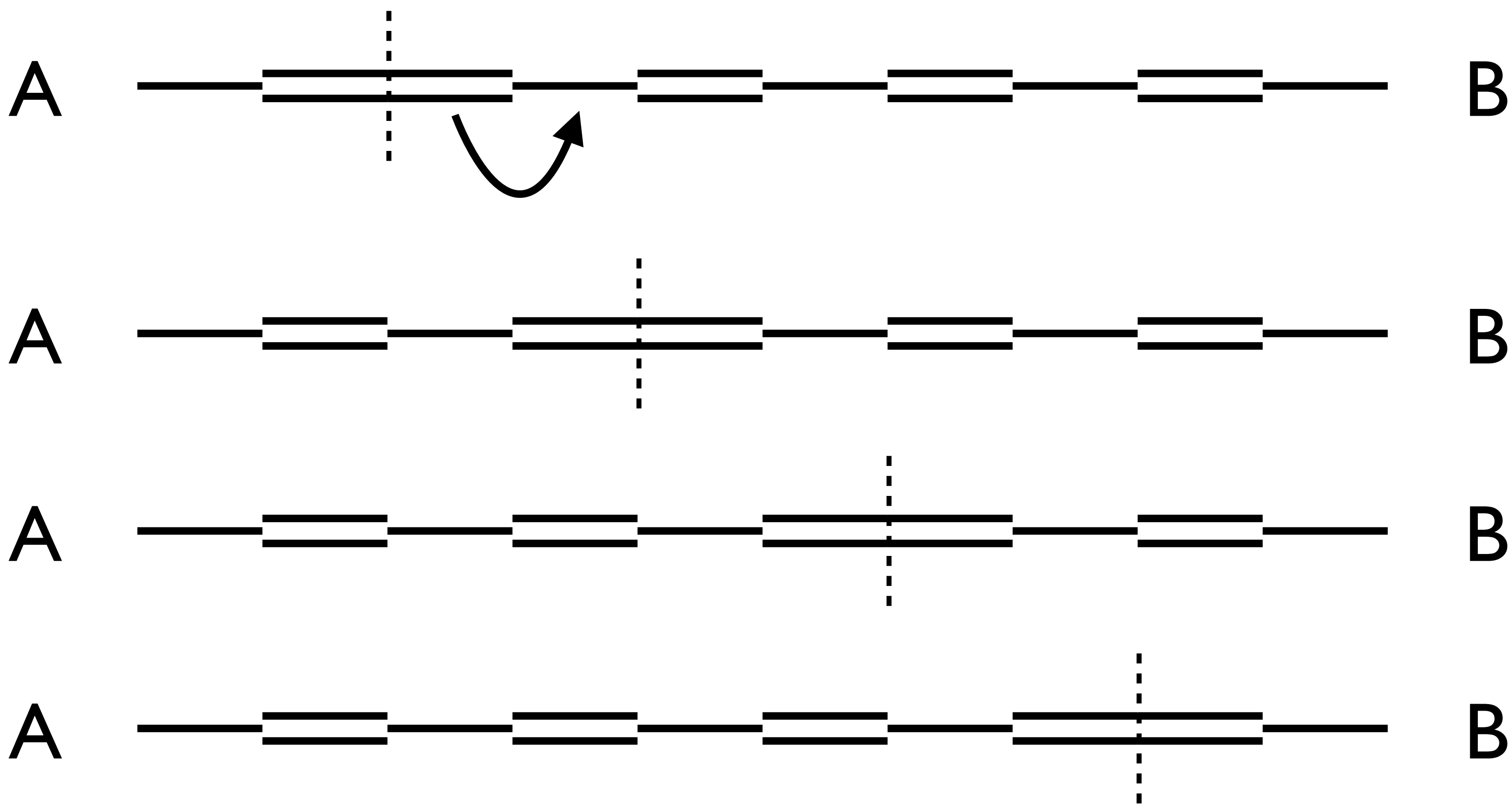

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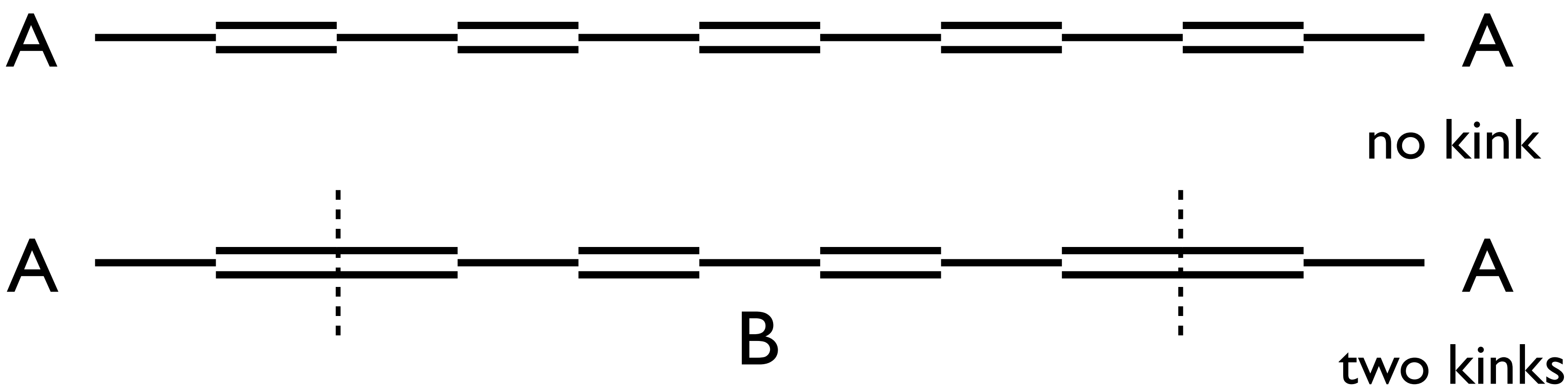

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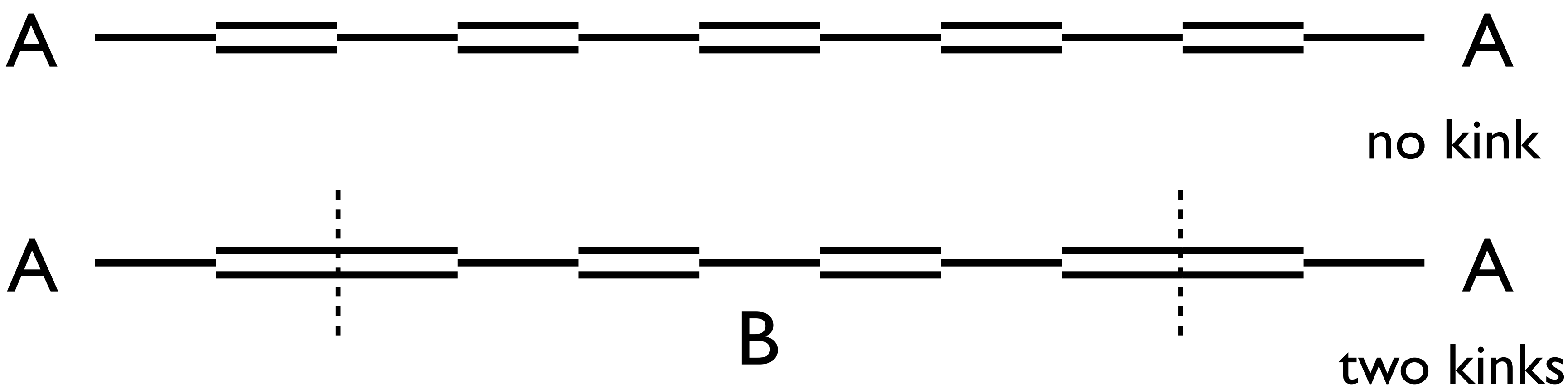

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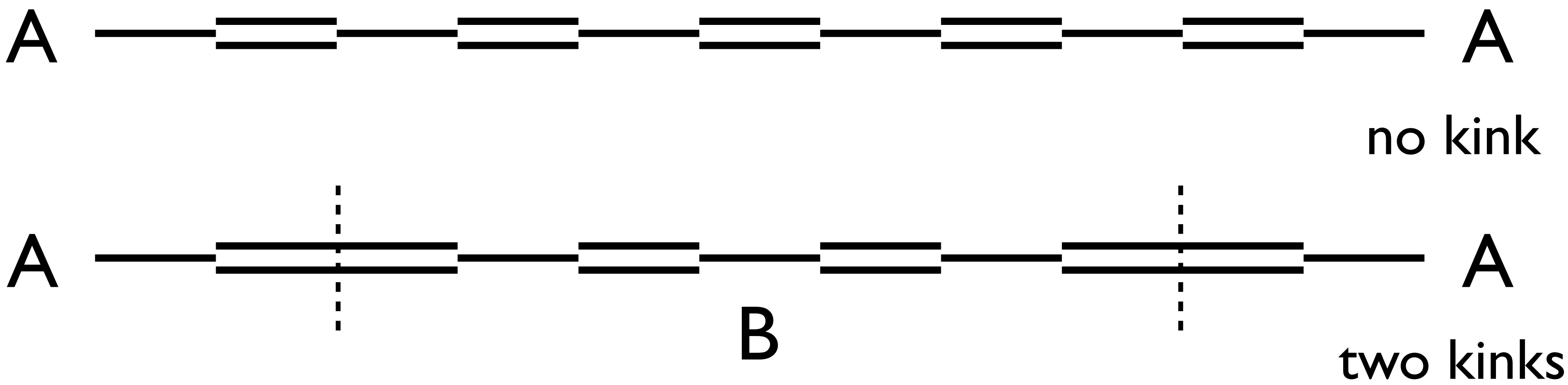

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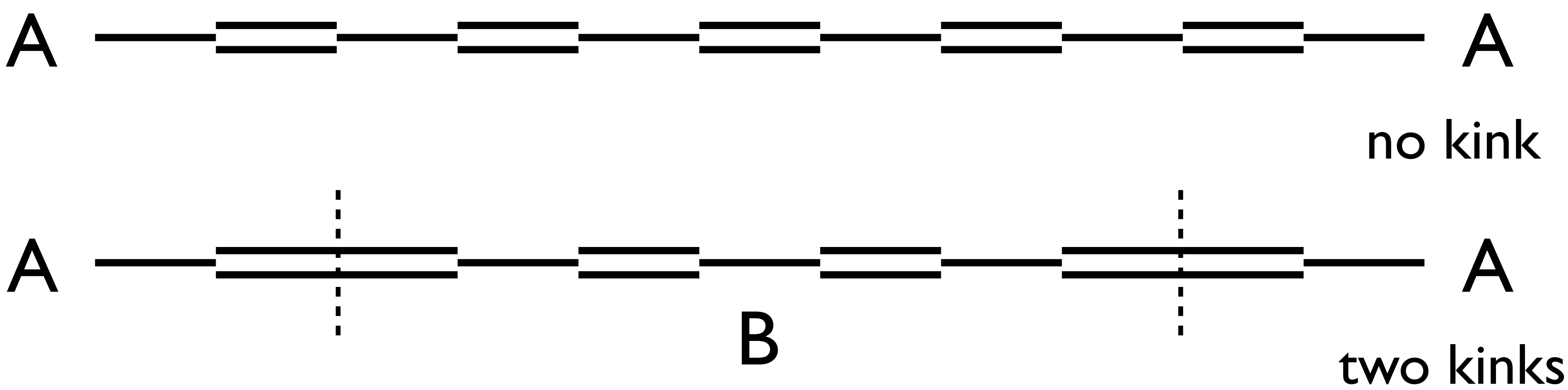

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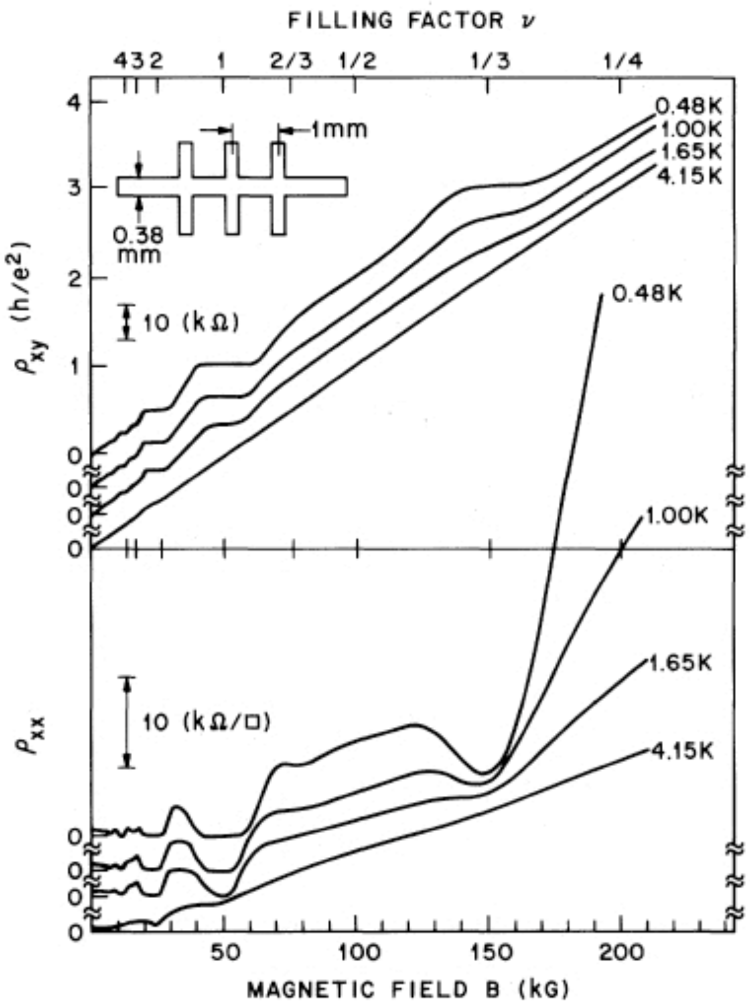
If the excitations have ‘smaller’ quantum numbers that the constituent particles, one speaks of ‘quantum number fractionalization’

Fractional quantum Hall effect

In 1982, the 'fractional' quantum Hall effect was discovered, namely, a plateau in the Hall conductance, with fractional value:

$$\sigma_H = \frac{1}{3}\frac{e^2}{h}$$

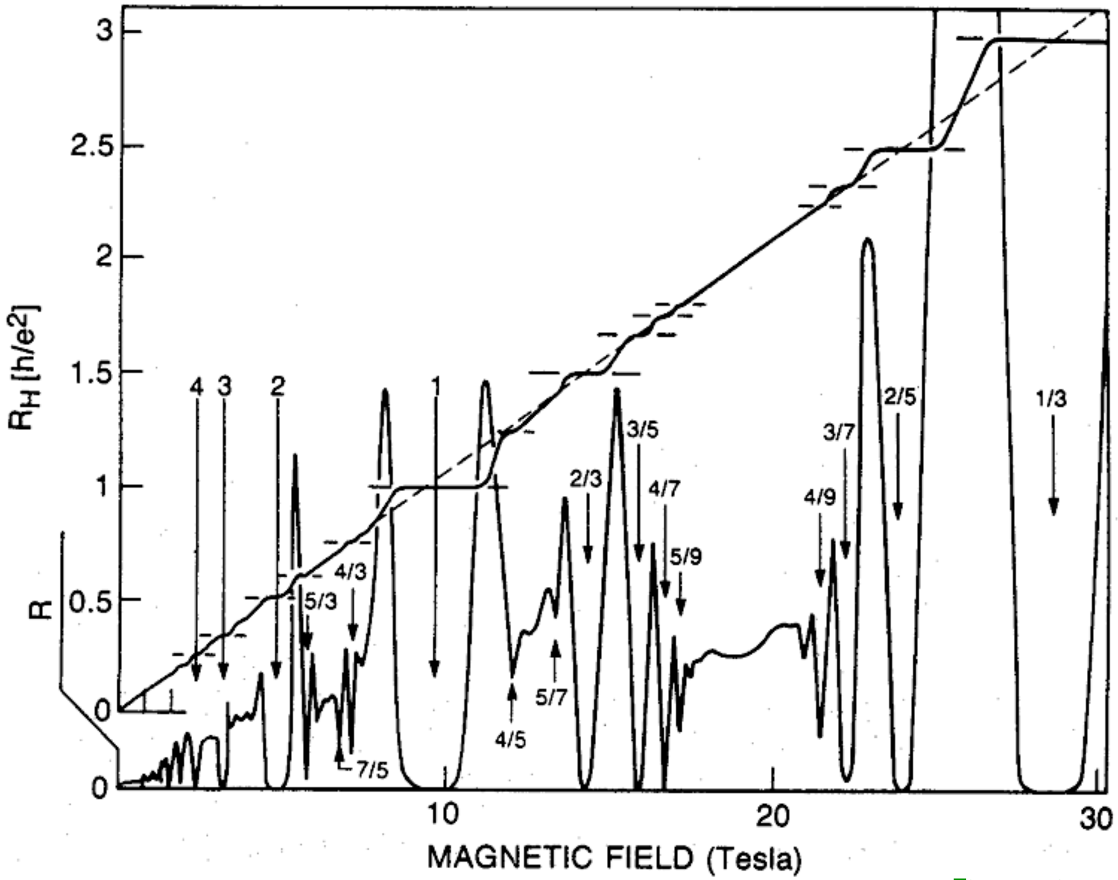


Tsui et. al, 1982

This effect was explained in 1983 by Laughlin. This state has particles with fractional charge (e/3) and so-called 'fractional statistics', they are neither fermions nor bosons!

Fractional quantum Hall effect

Many such fractional phases have been observed!



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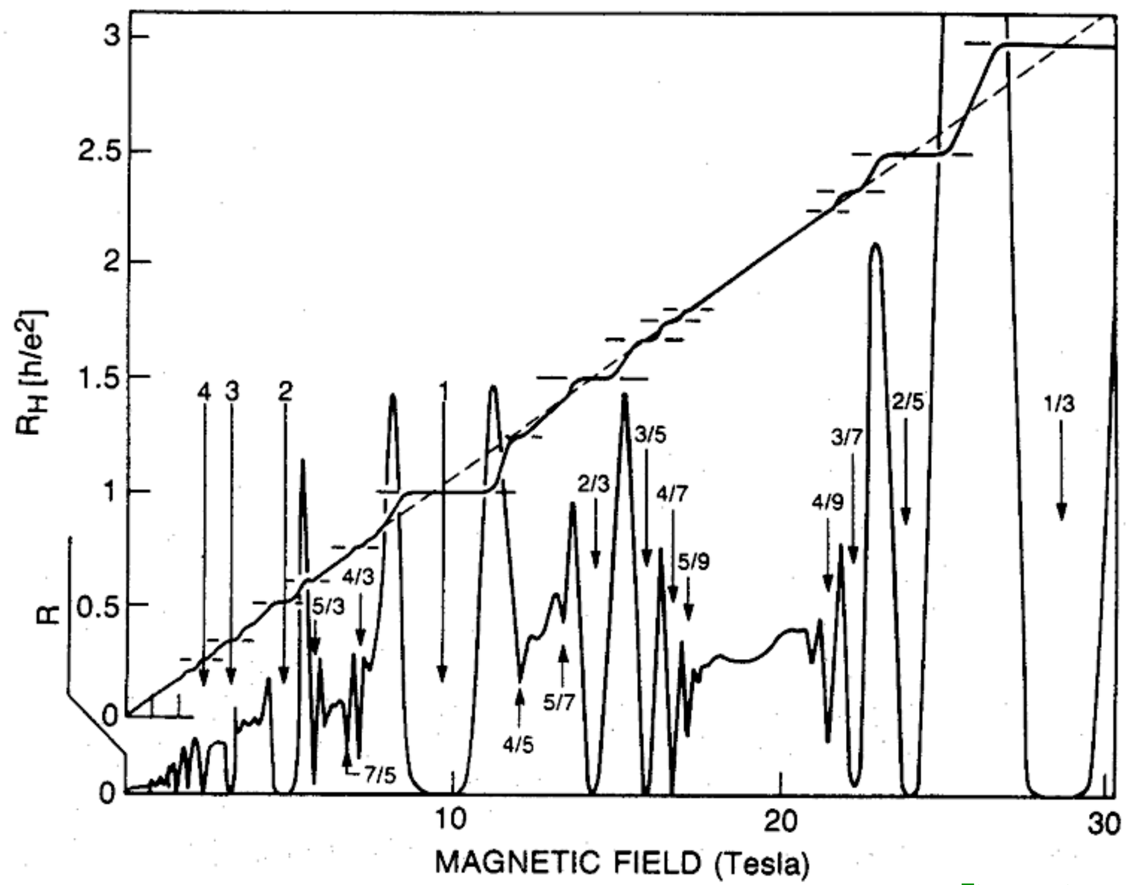

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Note: odd denominator filling fractions (electrons are fermions)

Measuring the fractional charge

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In the quantum Hall contacts, one let two edges come close, so that the particles can ‘tunnel’ from one edge to the other.

The noise in the tunneling current is proportional to the charge of the particles that tunnel!

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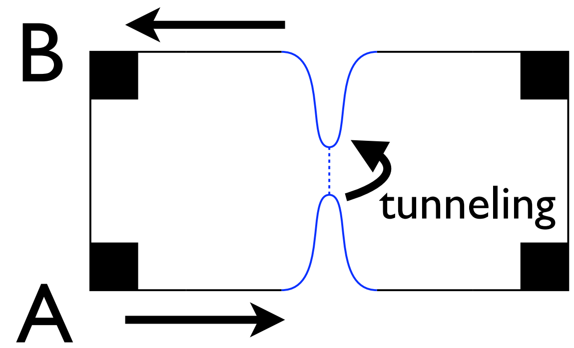
Schematic setup for a shot-noise experiment: Send in current via contact A, and measure the tunneling current at B



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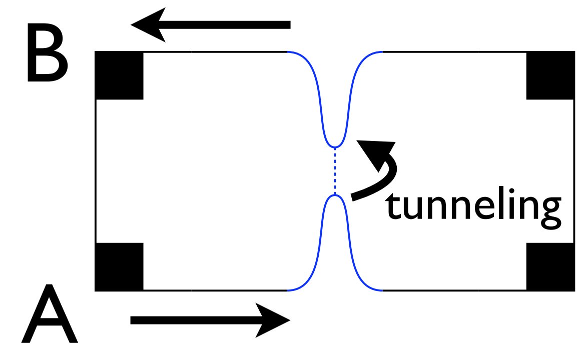

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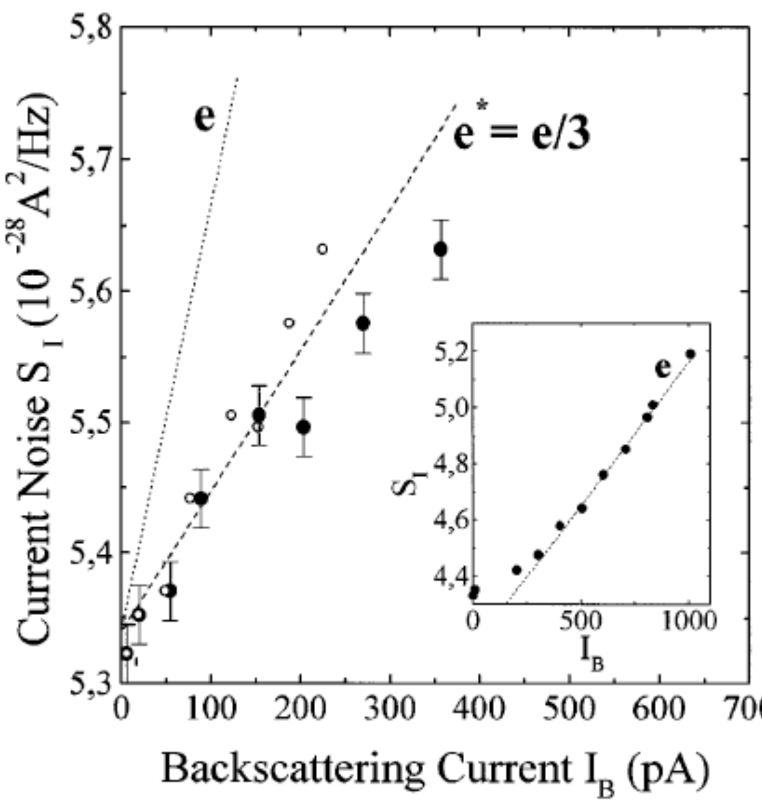
$$S = e^* < I_t >$$



Glattli, 1997

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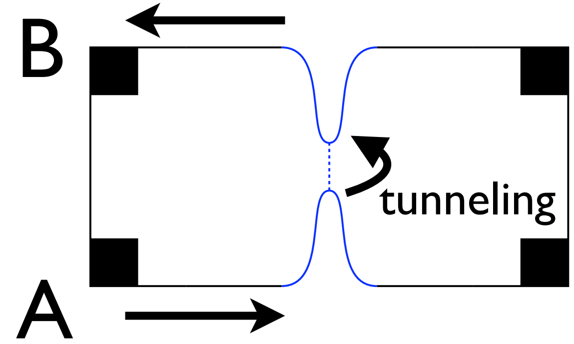

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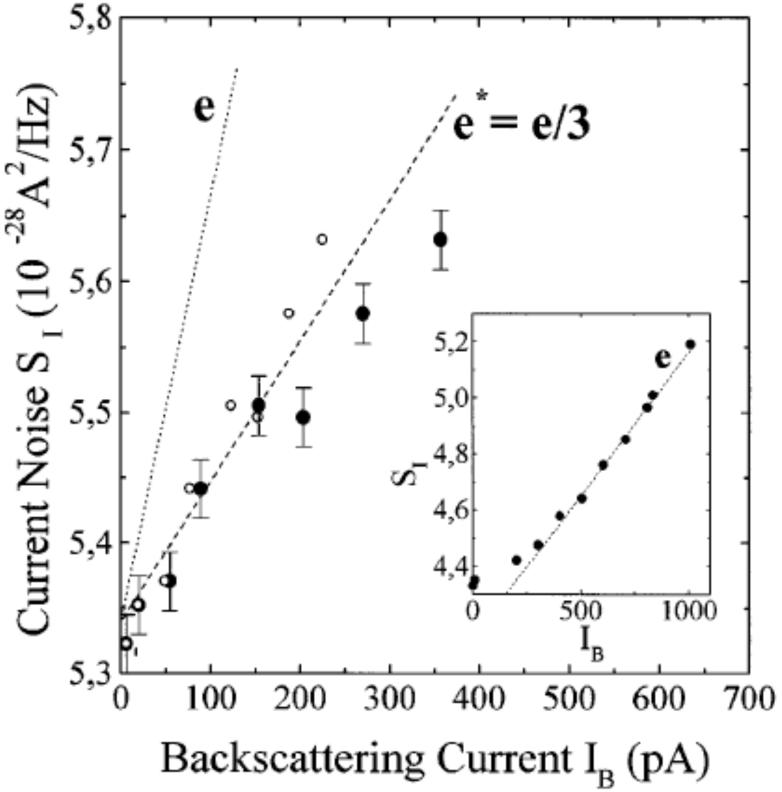
Several experiments, in various groups, have confirmed the existence of fractionally charged particles!



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Statistics of particles

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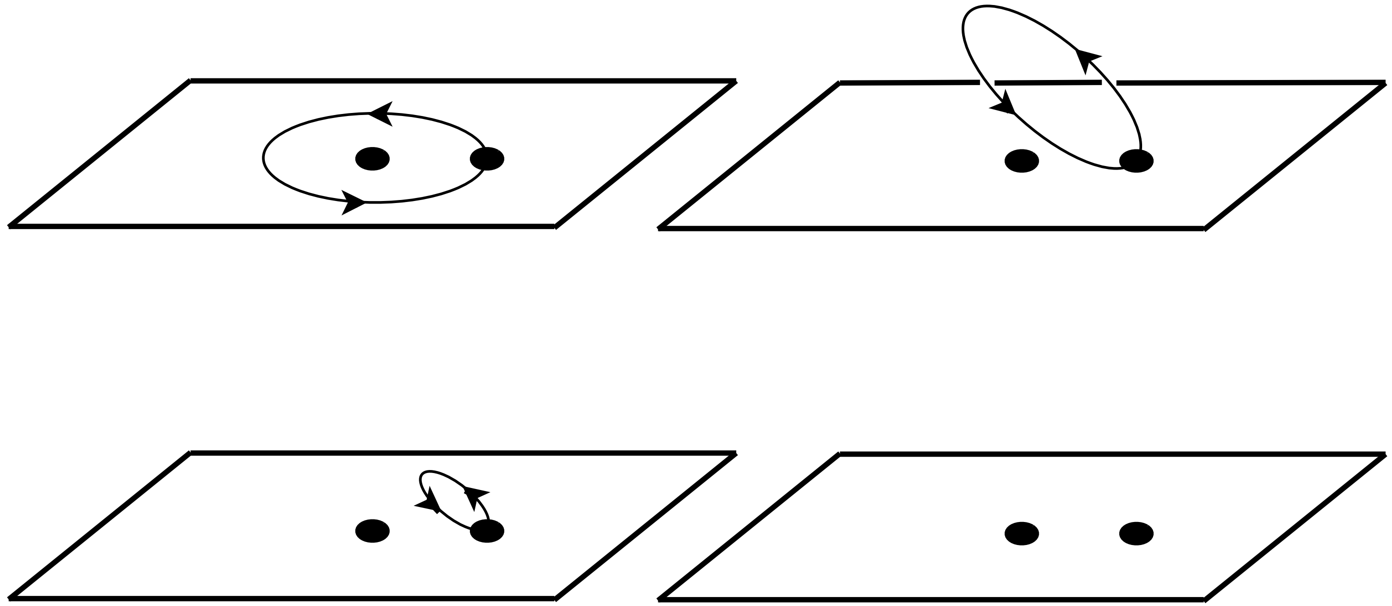

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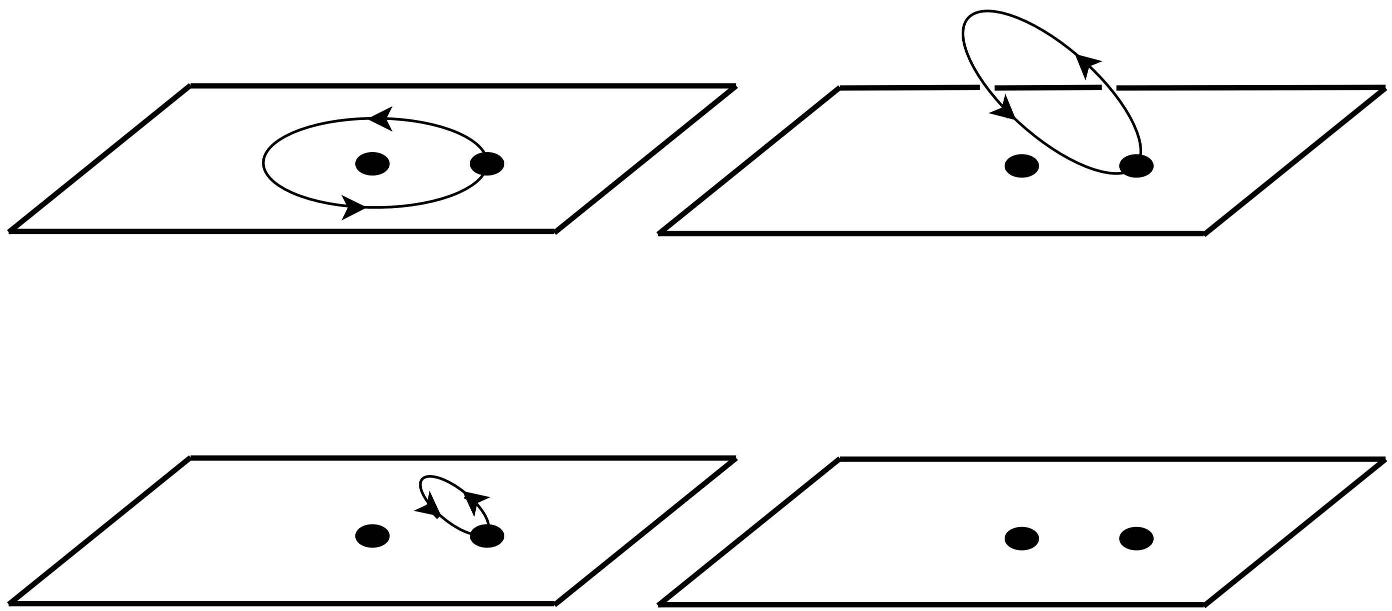

So the phase for a single exchange:
+ sign (bosons)
- sign (fermions)

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Detecting fractional statistics has proven to be much harder than fractional charge. Attempts using interferometry have been made by several groups.

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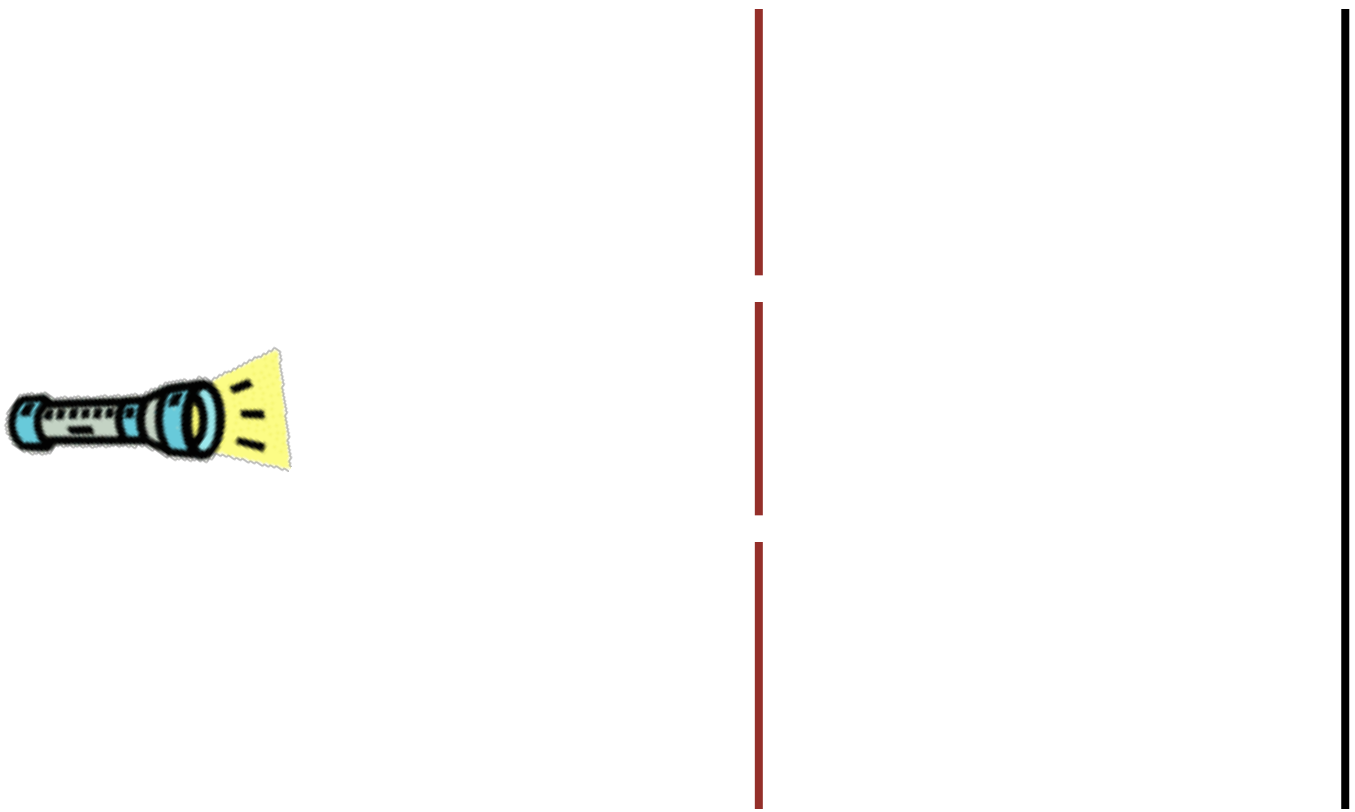

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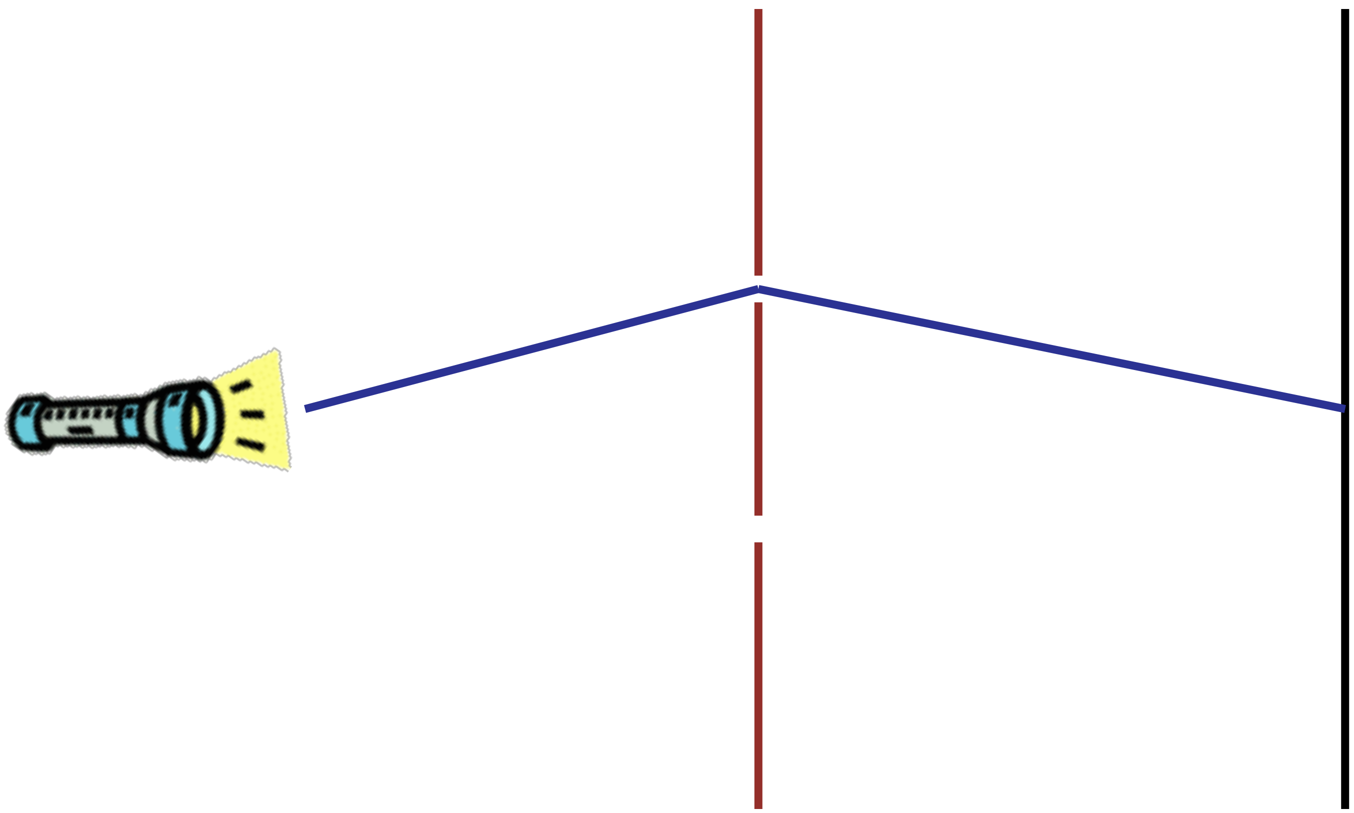

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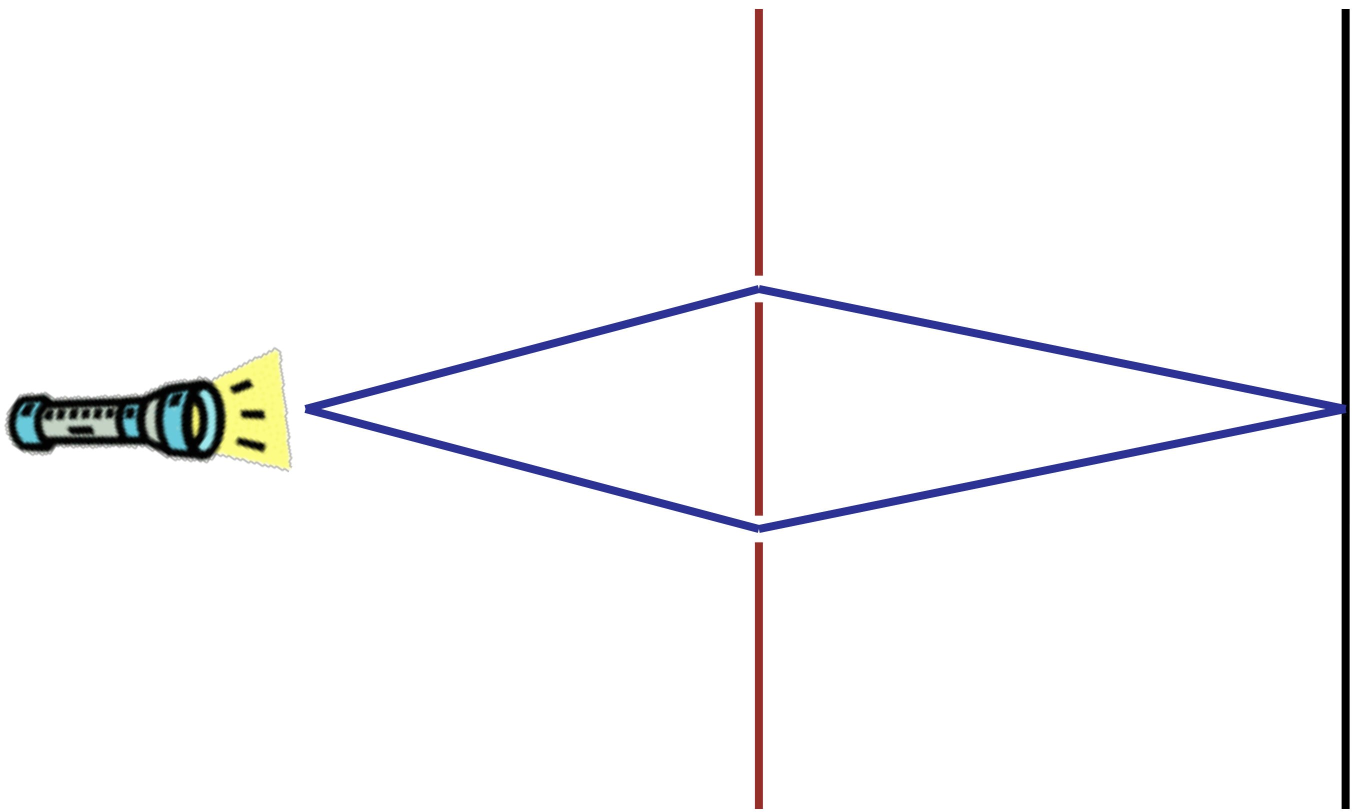

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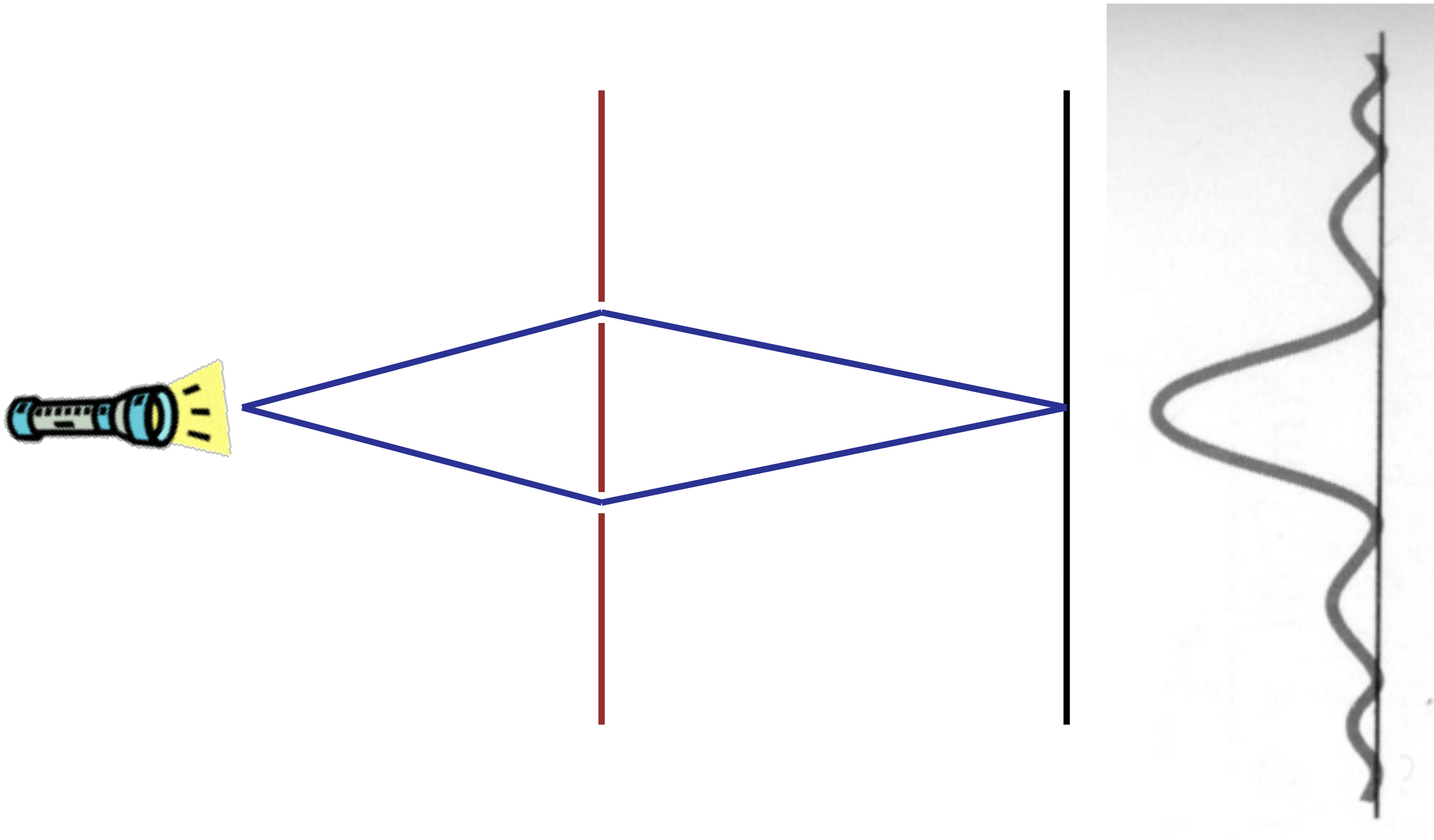
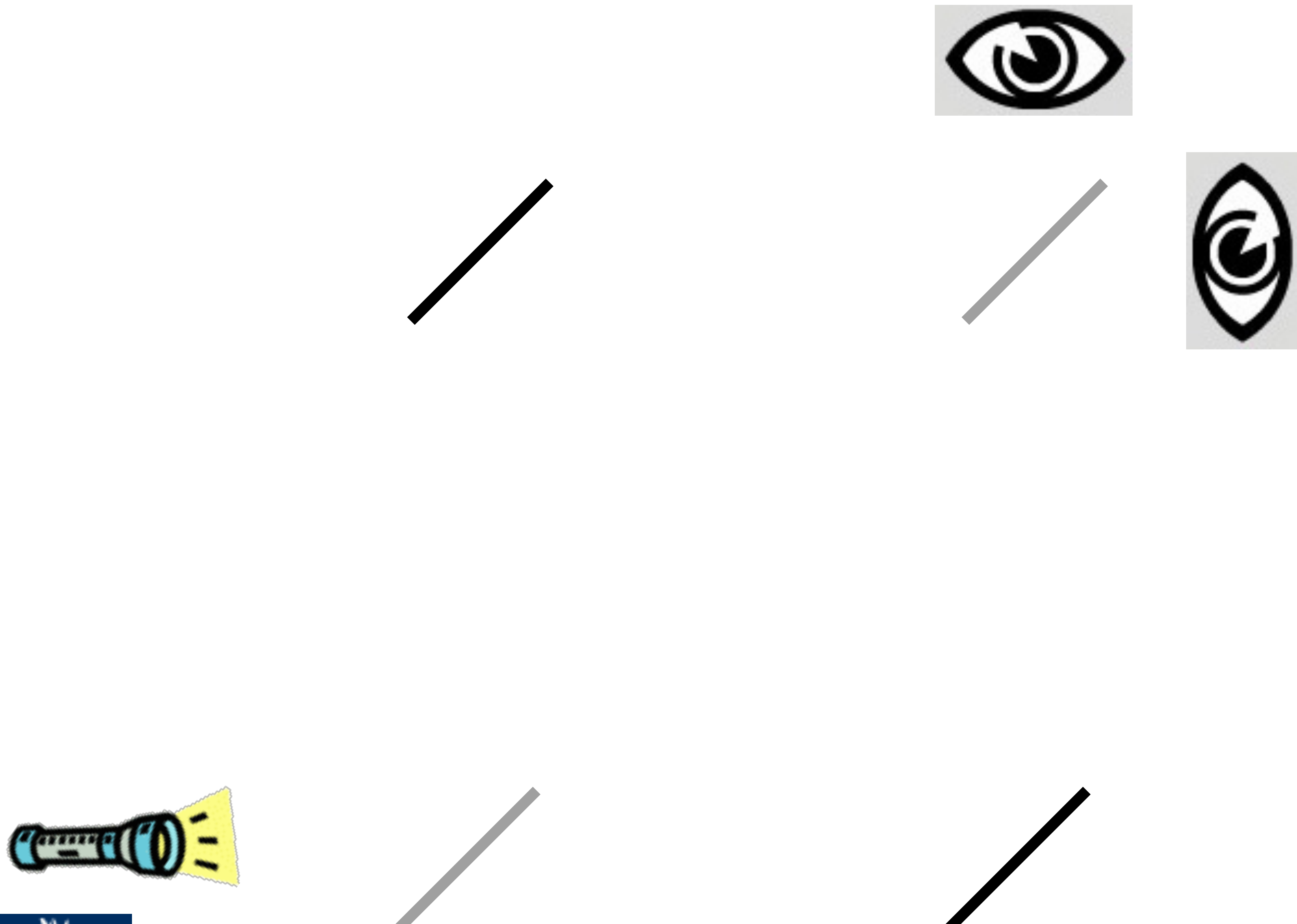


Mach Zehnder interferometer



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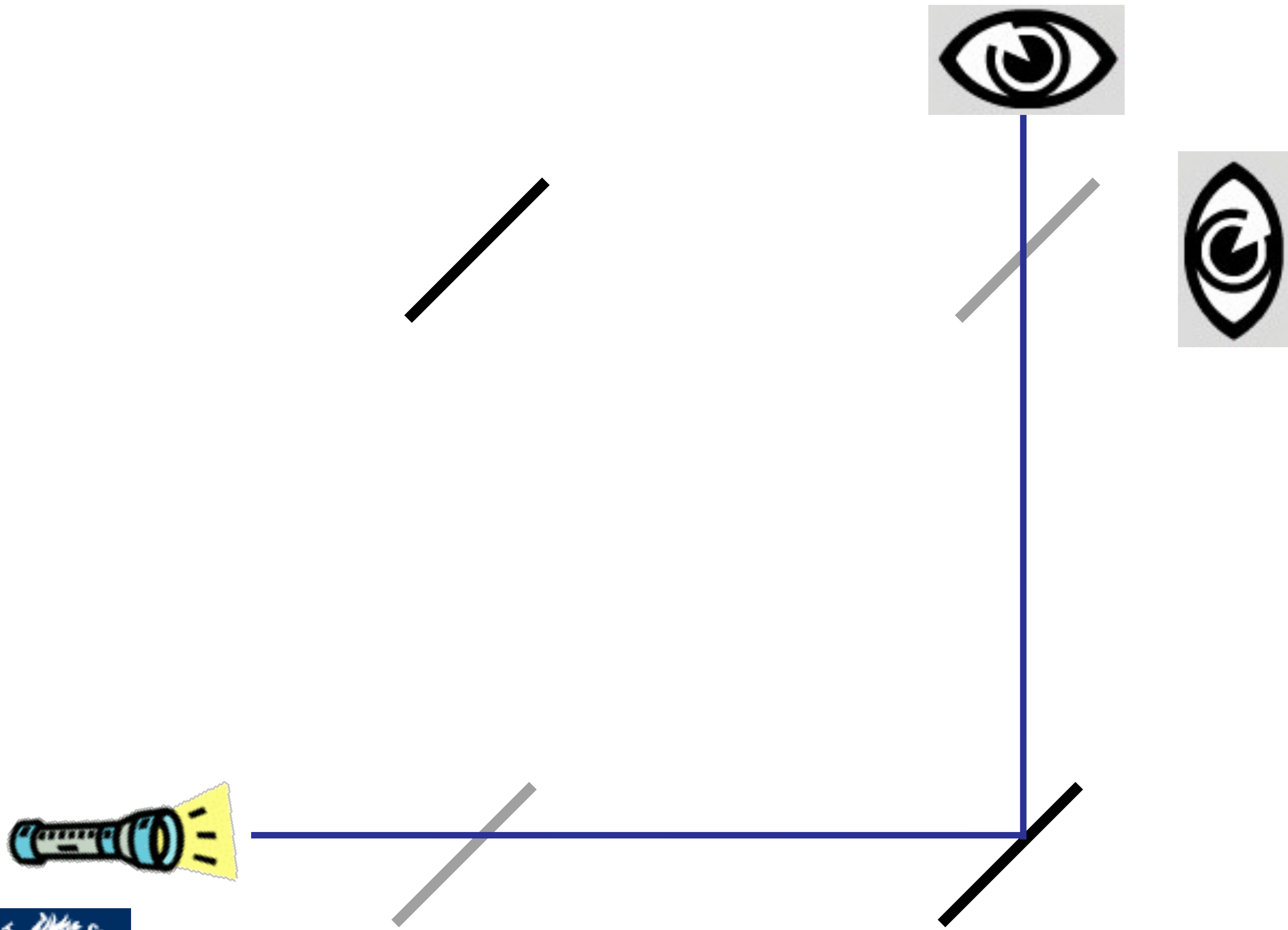

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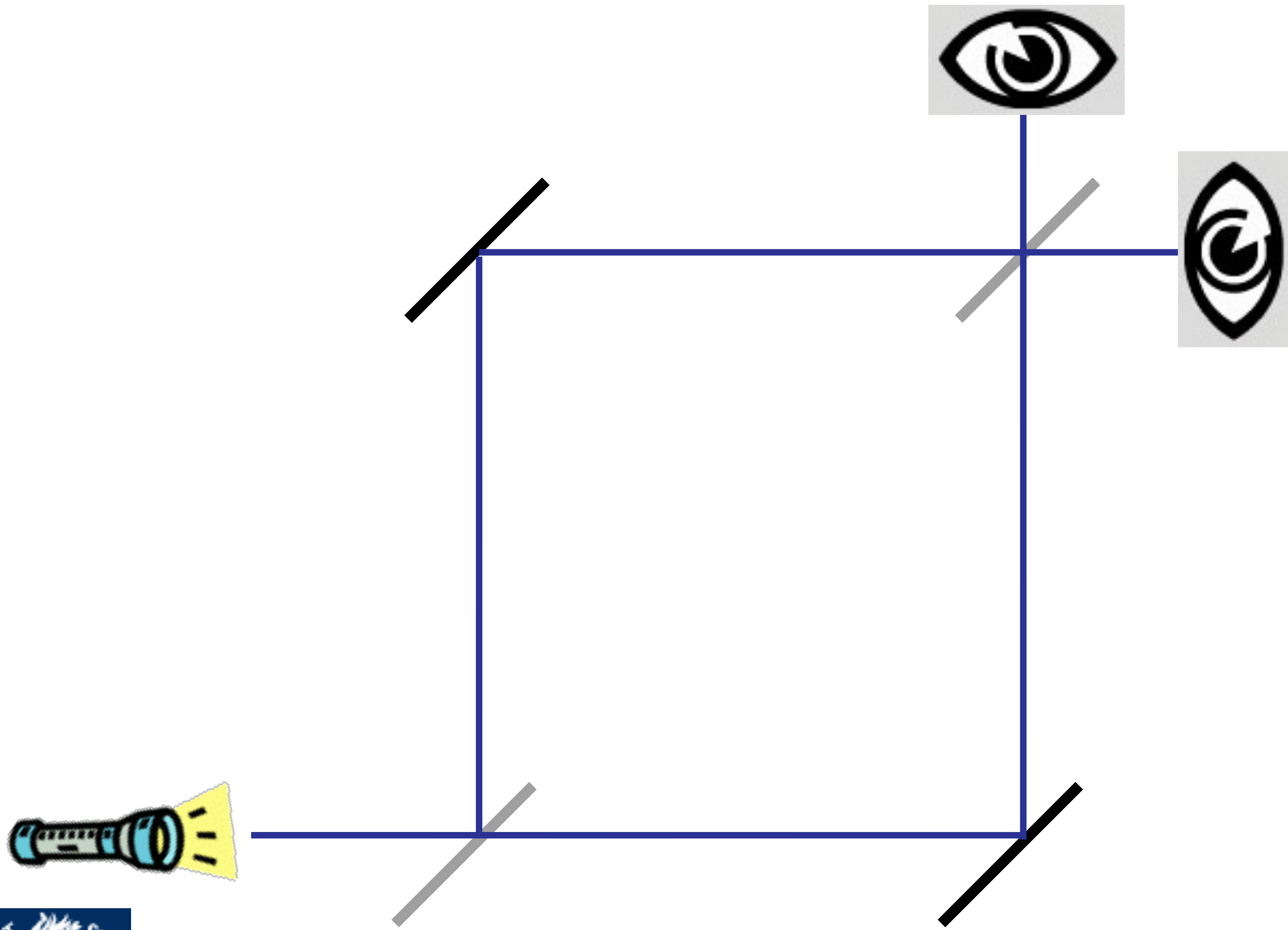

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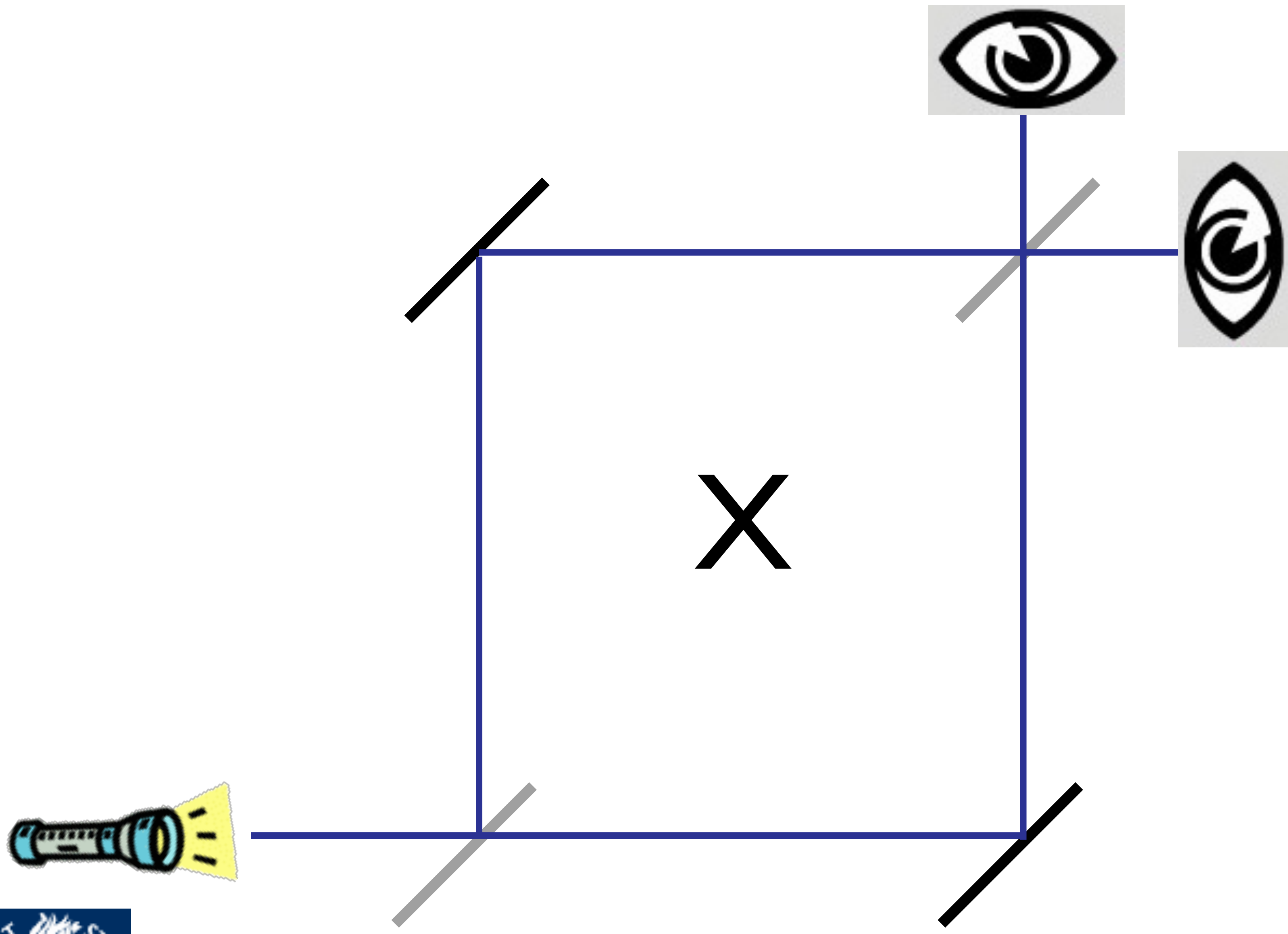

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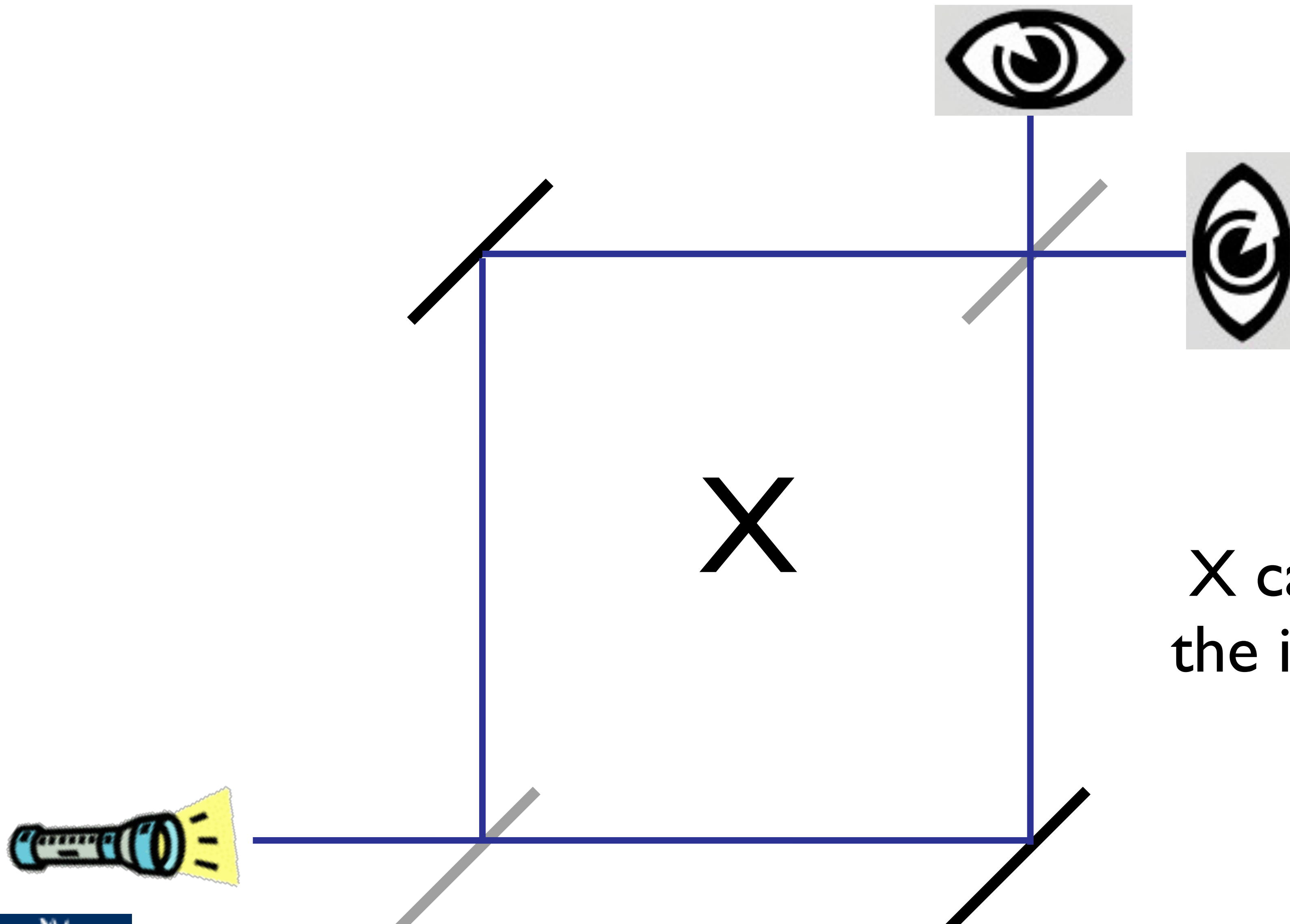
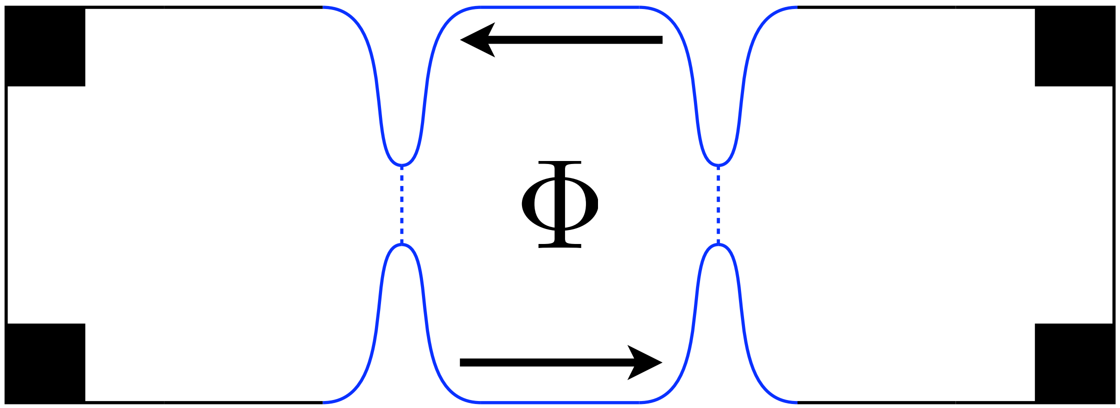


X can influence
the interference!

Interference in the quantum Hall effect



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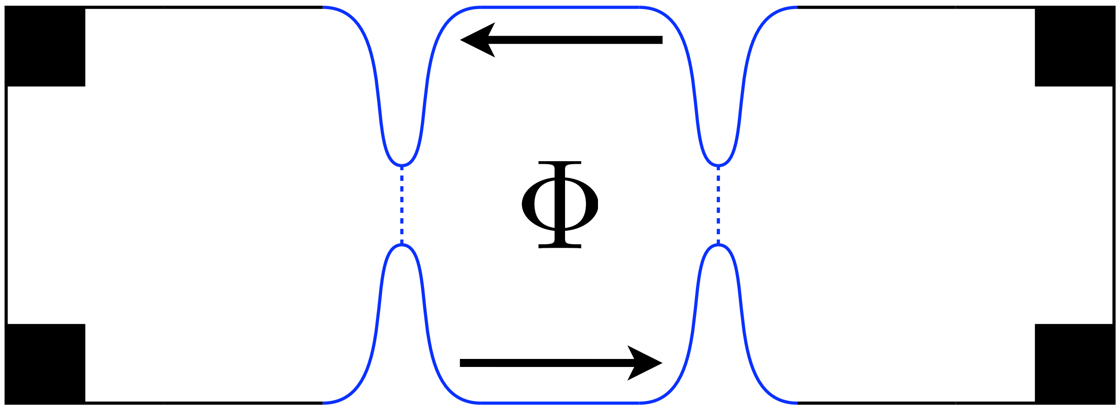
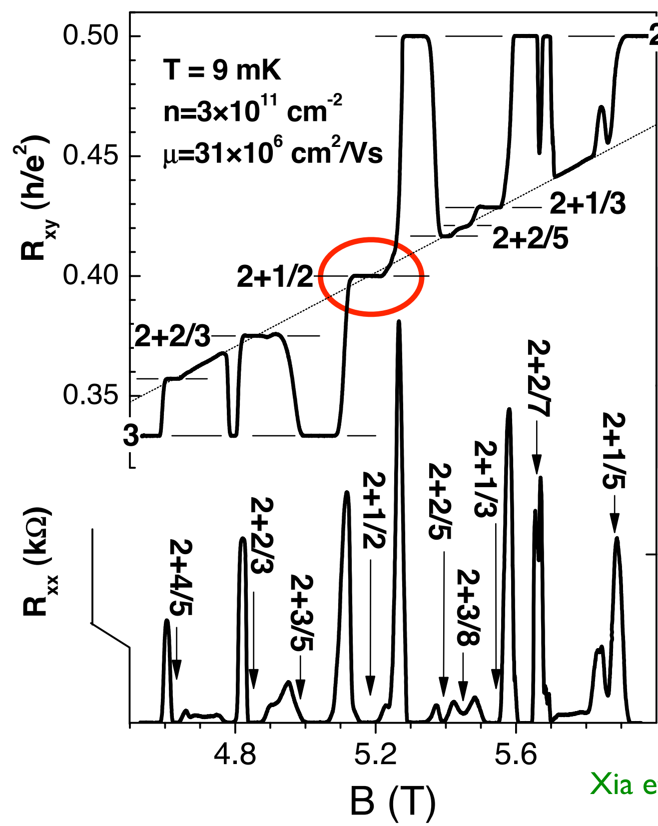


In the quantum Hall effect, the particles move along the edge. In the constrictions, the particles can tunnel from one edge to the other.

To probe statistics, a double point contact setup is an ideal setup, because the two path can interfere with each other, leading to so-called Aharonov-Bohm oscillations.

More exotic statistics?

In 1987, a quantum Hall state with an even denominator Hall conductance was observed!



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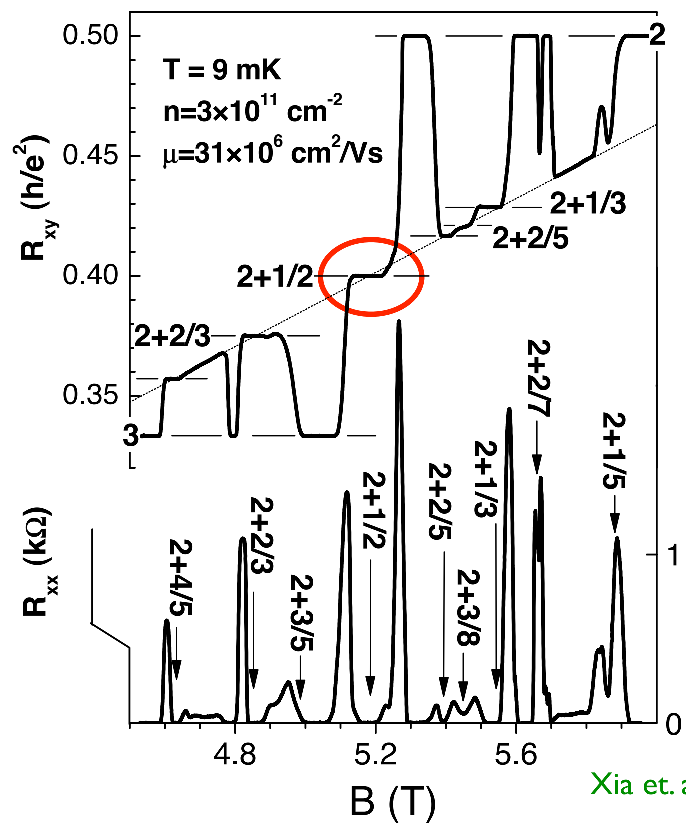

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In 1990, Moore and Read proposed an extraordinary explanation:
a state with anyons that have charge e/4 and exhibit *non-abelian* statistics!

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One has to perform topologically non-trivial operations, such as braids!

Braiding non-abelian anyons

The ‘wave function’ of four anyons can be described as follows

$$\Psi(w_1, w_2, w_3, w_4) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

The exchange of two anyons is described by a matrix, not just a phase!

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Changing the order of the exchanges, gives a different result!

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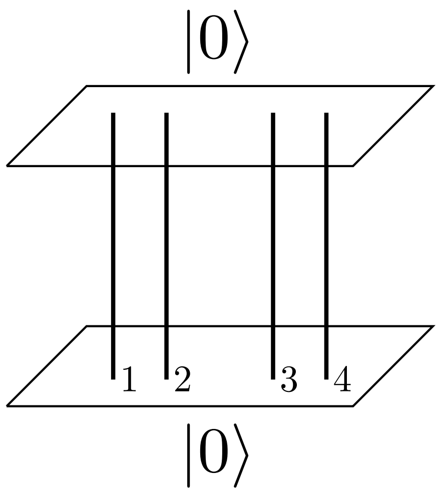

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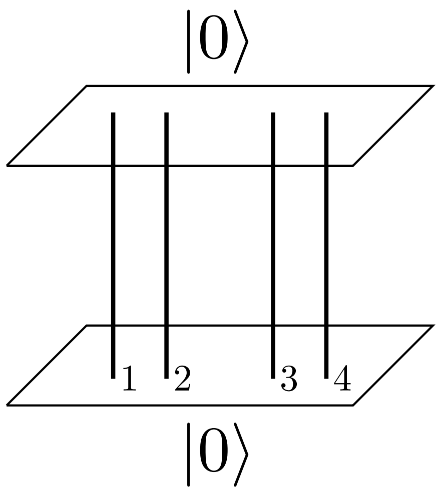

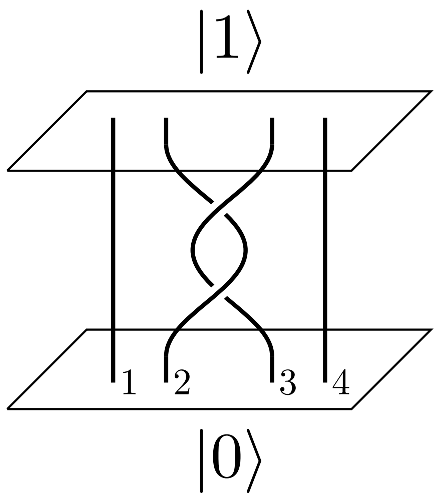

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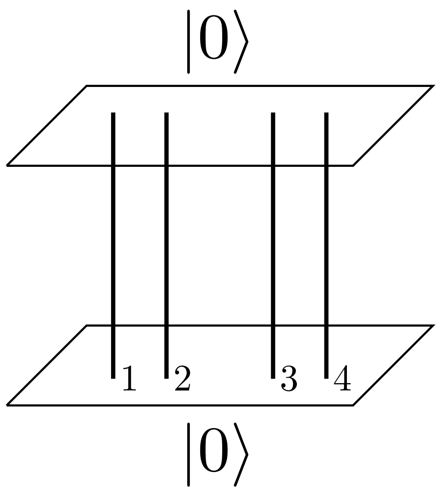
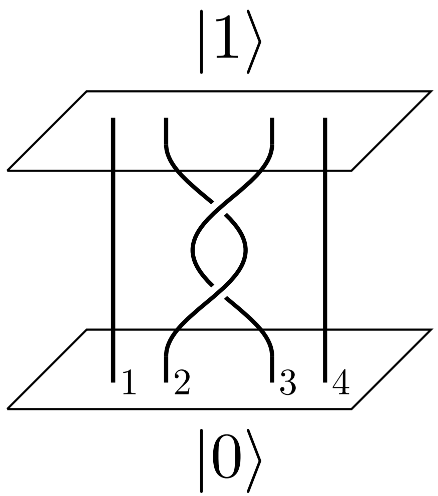
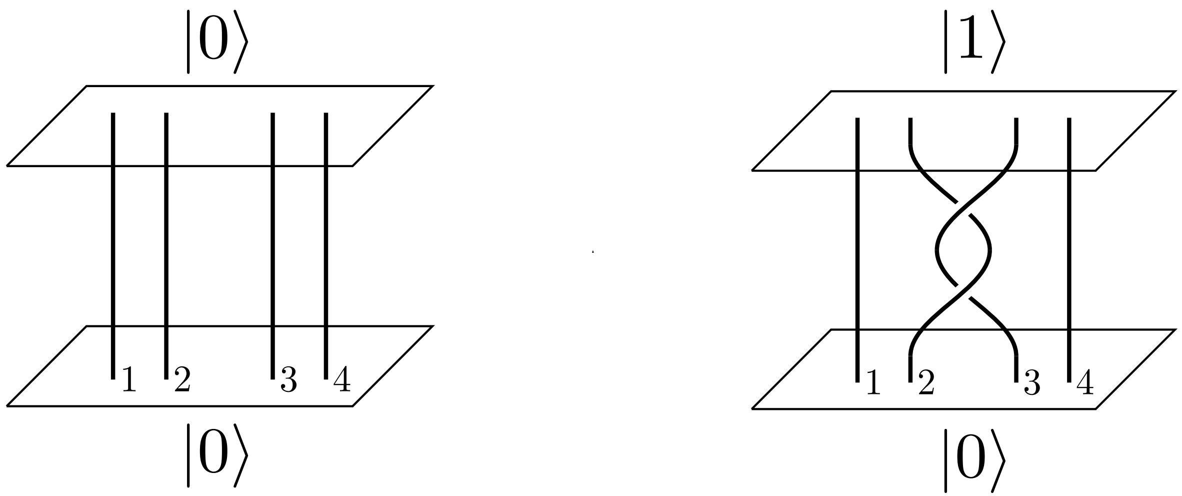




The result does not depend on the details of the exchange.
Again, we have topological protection!

Braiding non-abelian anyons

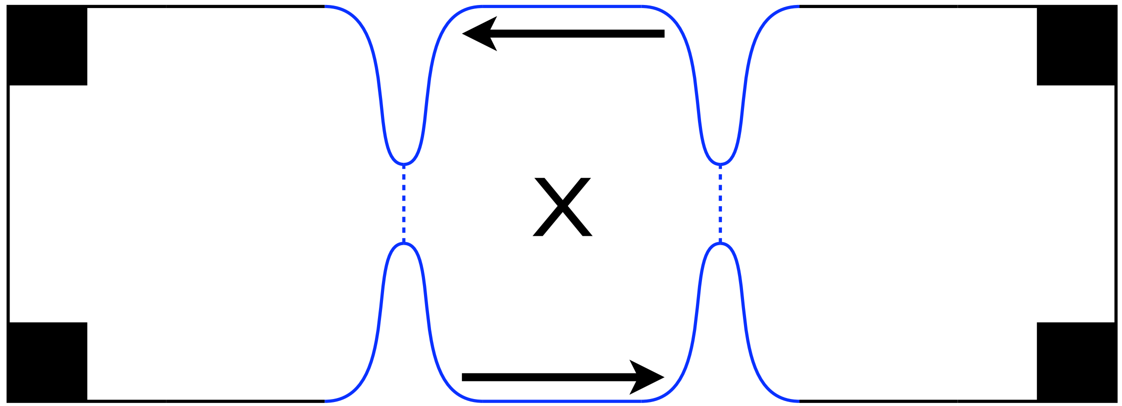
Braiding the anyons in the Moore-Read state: implements a NOT gate!



The result does not depend on the details of the exchange.
Again, we have topological protection!

If one uses this in a computation, one has to perform a measurement in the end. Interferometry can be used for this!

Interferometry in the Moore-Read state

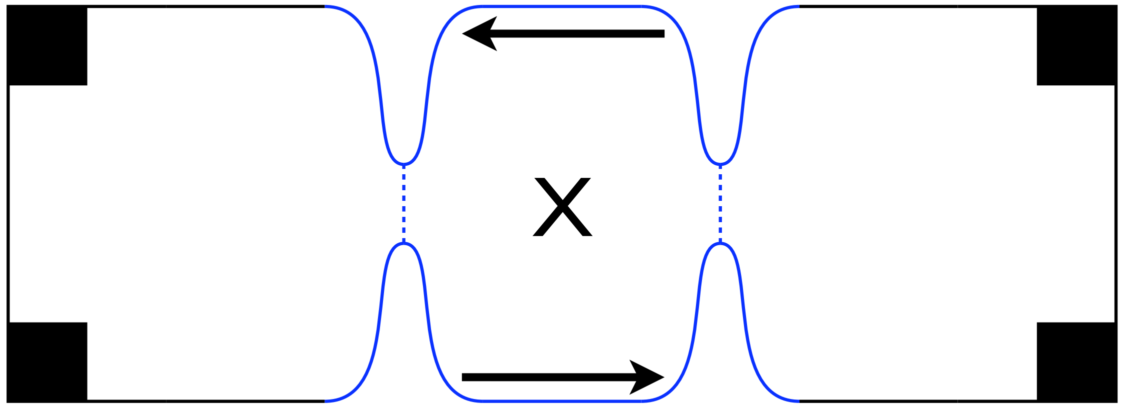


The interference pattern depends on how many anyons are in the interferometer region.

For an odd number of anyons, the interference vanishes completely. This can (in principle!) be used to measure the outcome of the computation.

There are promising experiments, but they are under (heavy) debate.

Interferometry in the Moore-Read state



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Fradkin et al., 1998

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Bonderson et al., Stern et al.,2006

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Topological quantum computation?

Topological states with non-abelian anyons are rare, and hard to control.

But, recently, similar anyons (Majorana bound states) might have been observed in one-dimensional nano-wires. This could give better control on the system.

The anyons of the Moore-Read state are not universal, the phase gate can not be constructed in a topological protected way.

States that are rich enough might exist (with conductance 12/5), but no experiments have been done to verify this (really hard!)

Topological states are a very interesting alternative to quantum computation, because of there inherent robustness, but the field is much behind other approaches.

Conclusions part II

In topological phases, particles with fractional quantum numbers can sometimes appear!

Fractionalized particles have been predicted to exist in several systems, quantum Hall effect, on surfaces of topological insulators, in nano-wire systems (actually, they seem hard to avoid).

Experiments have seen signatures, that point that such particles really exist. More conclusive experiments are necessary!

If (once?) they are found, there are many ways to exploit them in very interesting devices (theory is way ahead of experiments here)!