What is topological matter & why do we care?

Part 1: what are topological insulators?

Eddy Ardonne
thanks to:
Hans Hansson

WfSW:QT
2014-08-29

Topological insulators in the media!

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So, who knows what a topological insulator is?

Condensed matter physics

What phases of matter do exist?

How does matter go from one phase to another?

Daily life example: water and ice

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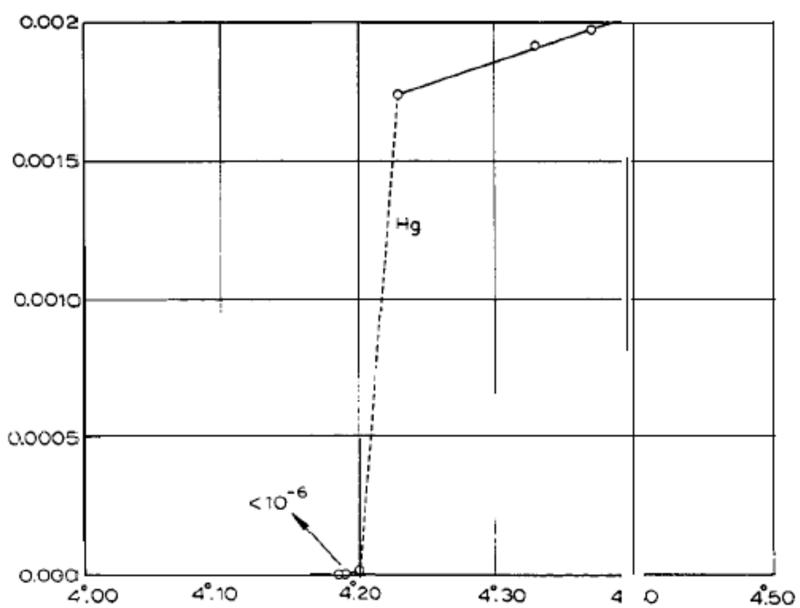

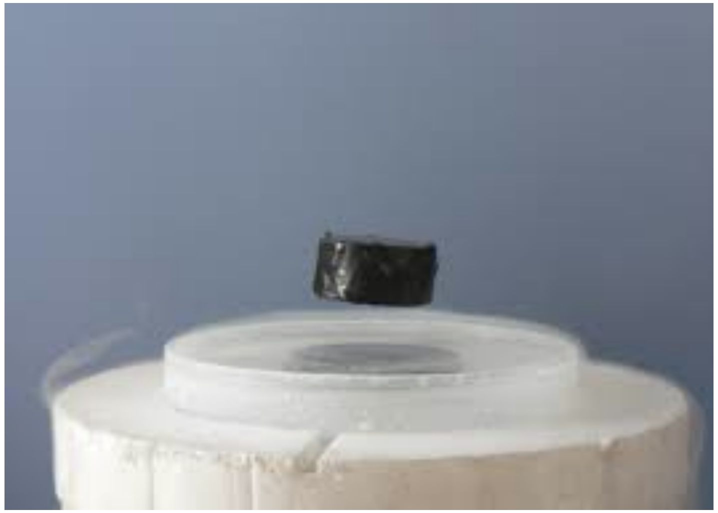

Kamerling Onnes et al., 1911

Condensed matter physics

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Some materials (such as wood) are insulating, they do not conduct current:

Condensed matter physics

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Some materials (such as wood) are insulating, they do not conduct current:

Both conductors and insulators are important, but the behaviour in between is really interesting: ***semi-conductors***!

Between metals and insulators

Semiconductors can be used to make very interesting devices, transistors!

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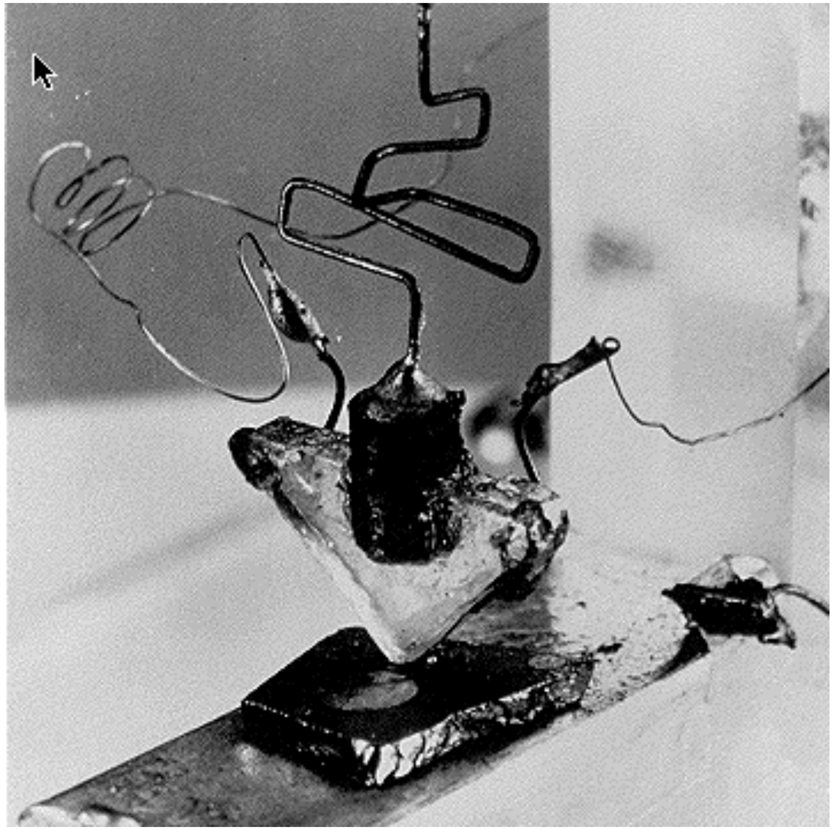
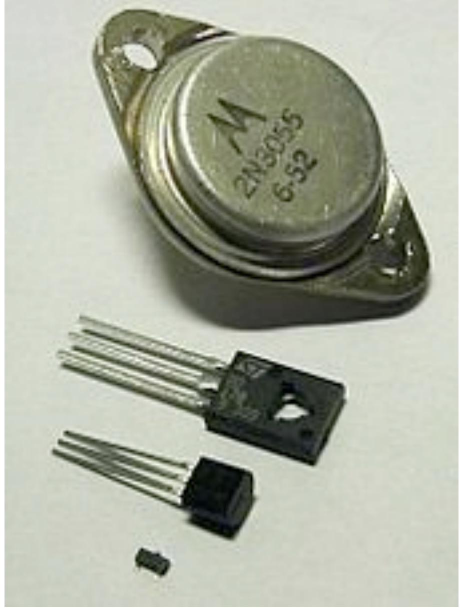


Bardeen et al., 1947



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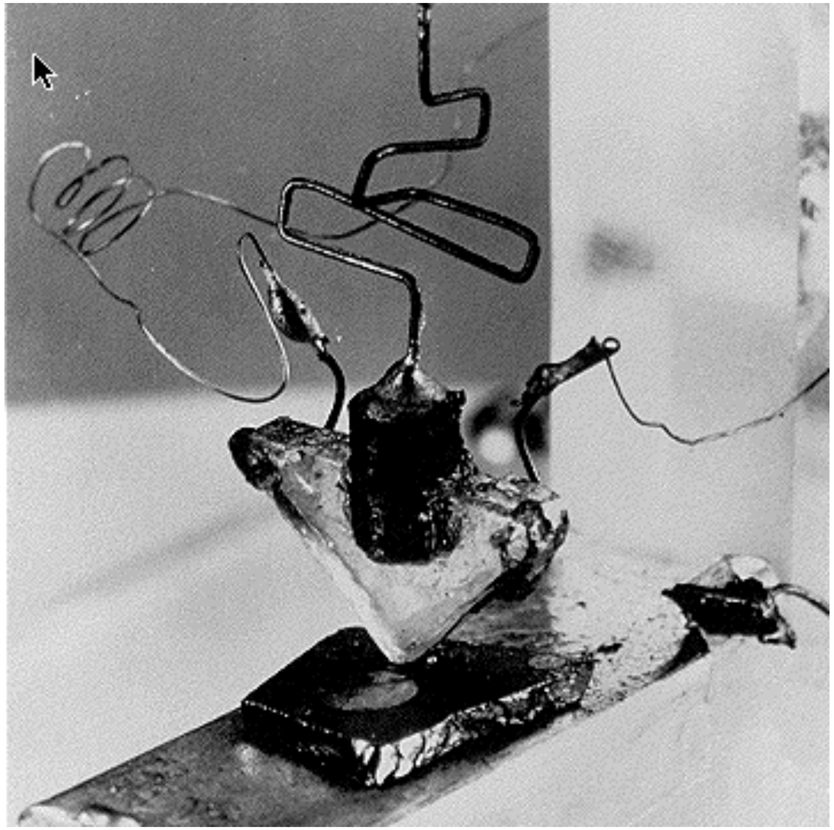

Bardeen et al., 1947

Impact on society is hard to quantify.

Topological insulators in words

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They are *insulating inside* (in ‘the bulk’)

They are *conducting on the surface*

The conductance on the surface is insensitive to dirt, disturbances, etc.

We say that the conductance is **protected** for **topological** reasons!

Crash course on topology

Crash course on topology

A football player would say:  $\neq$ 

Crash course on topology

A football player would say:  $\neq$ 

A topologist would say:  $=$ 

Crash course on topology

A football player would say:  ≠ 

A topologist would say:  = 

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Knot theory, some history

Inspired by Peter Tait's experiments on smoke rings, Sir William Thomson developed the idea that the different atoms are related to different **knots** (1867)!

The knots were thought to be different vortex rings in the aether. Tait started to classify all the different knots. However, the aether doesn't exist, so this idea failed.

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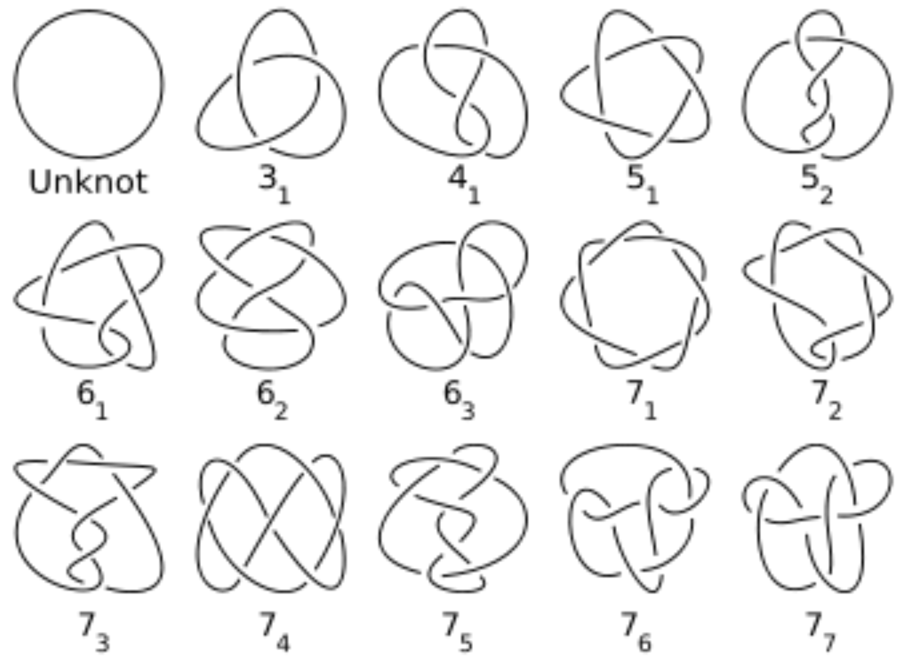
Knots play an important role in topological quantum computation!

Knots, some examples

For ‘small’ knots, it’s rather easy to tell them apart



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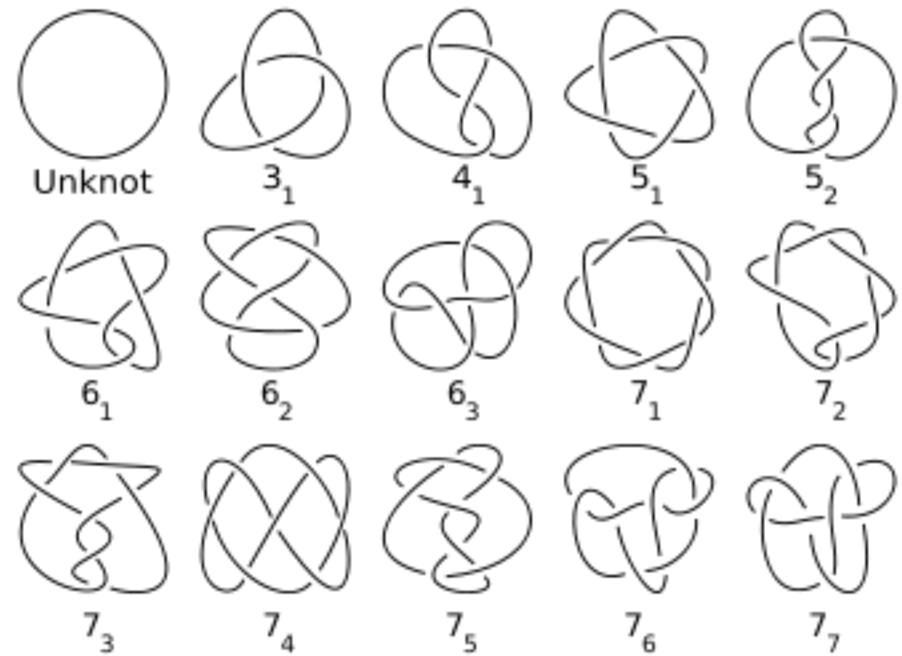
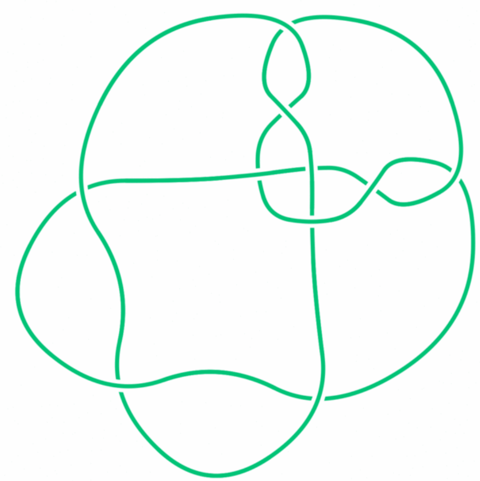
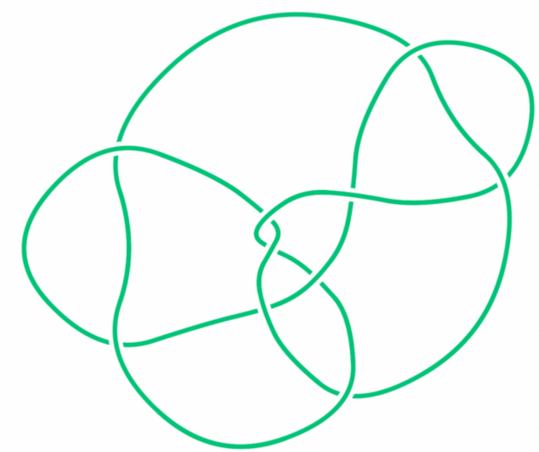


But are these the same or not?





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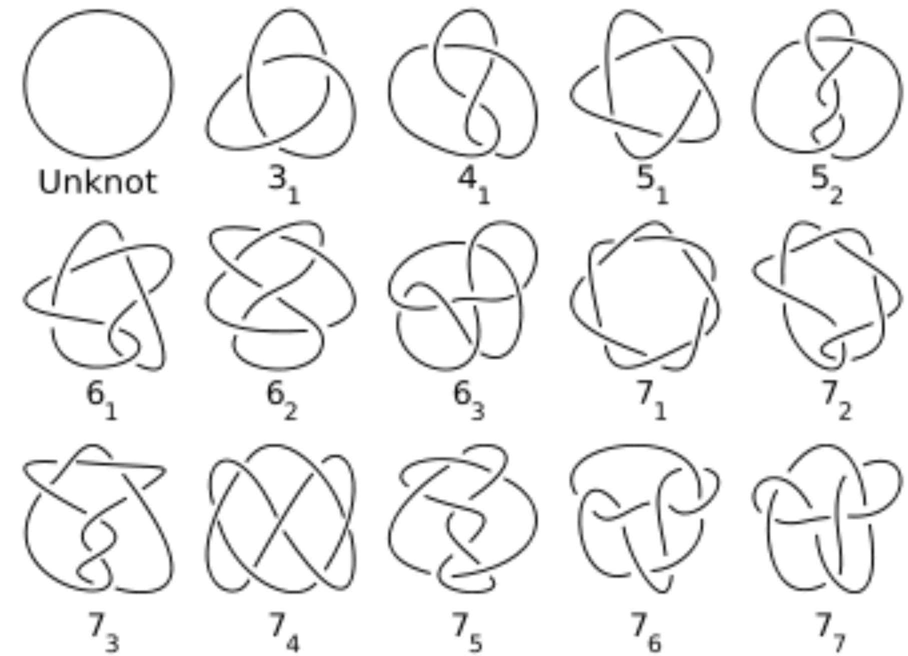

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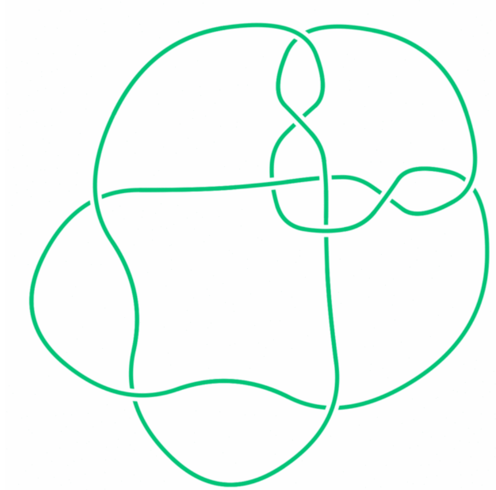

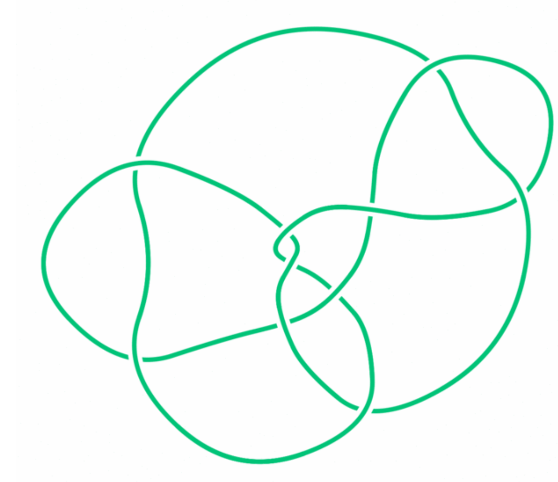

Yes, but for a long time they were listed as different in the literature!

Topological invariant: winding number

An example of a topological invariant is the winding number.
How many times winds a curve around the origin?

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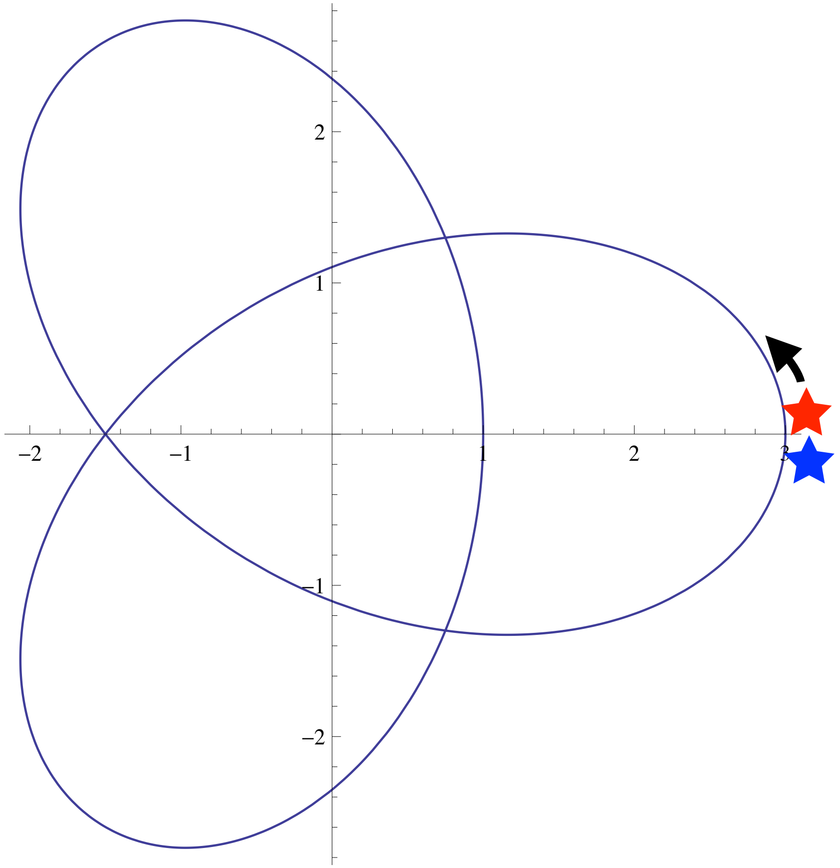

start
finish

winding number: +2

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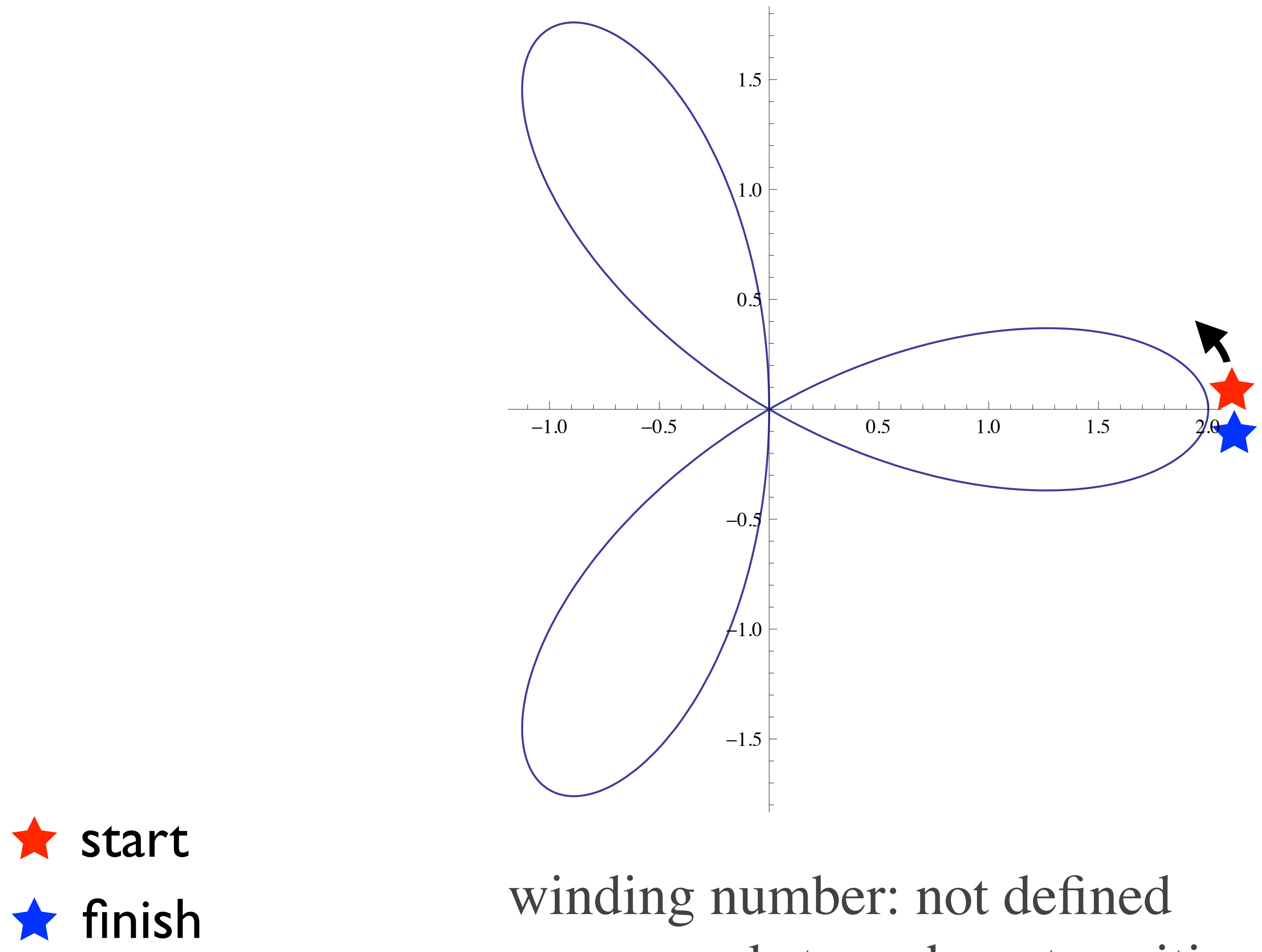

winding number: not defined
corresponds to a phase transition

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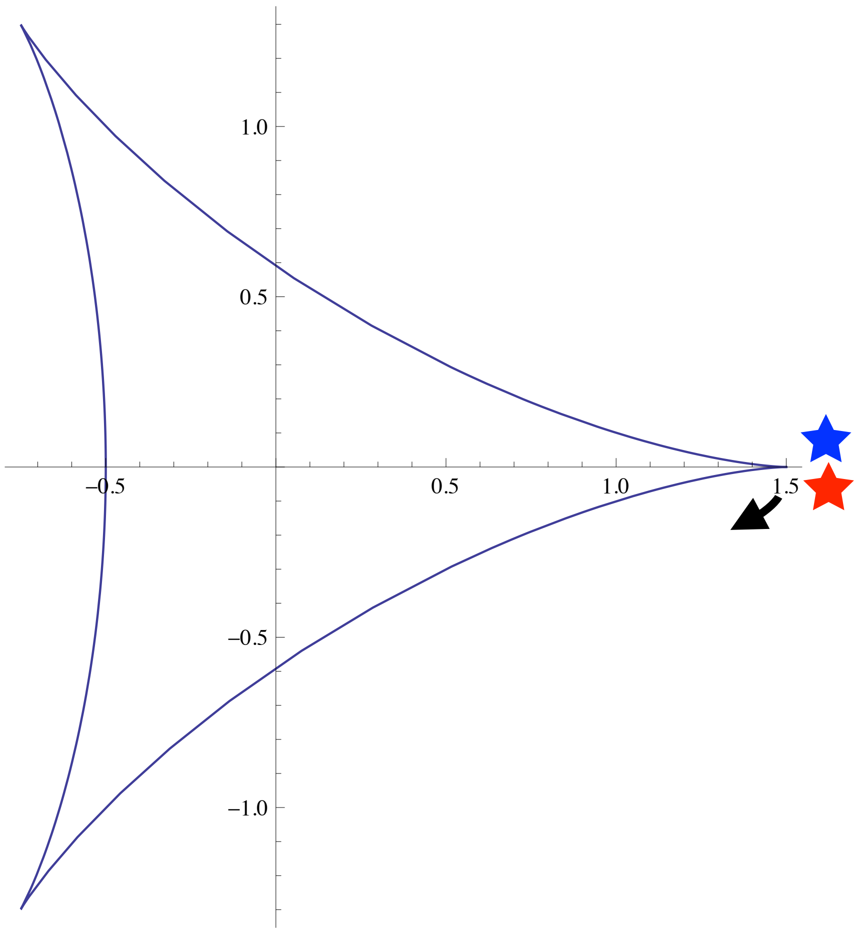

start

finish

winding number: -1

The quantum Hall effect

The first topological state that was observed is the ‘quantum Hall effect’, dating back to 1980.

The electrons are confined to a two-dimensional plane, between to semi-conductors. The quantum Hall effect occurs at very low temperatures, 1 Kelvin or lower, and in a strong magnetic field, 10 Tesla.

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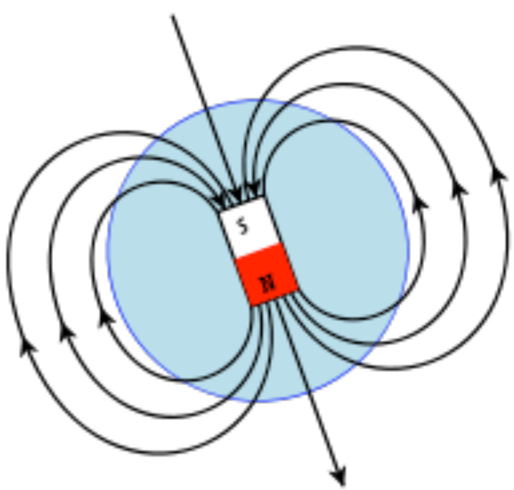
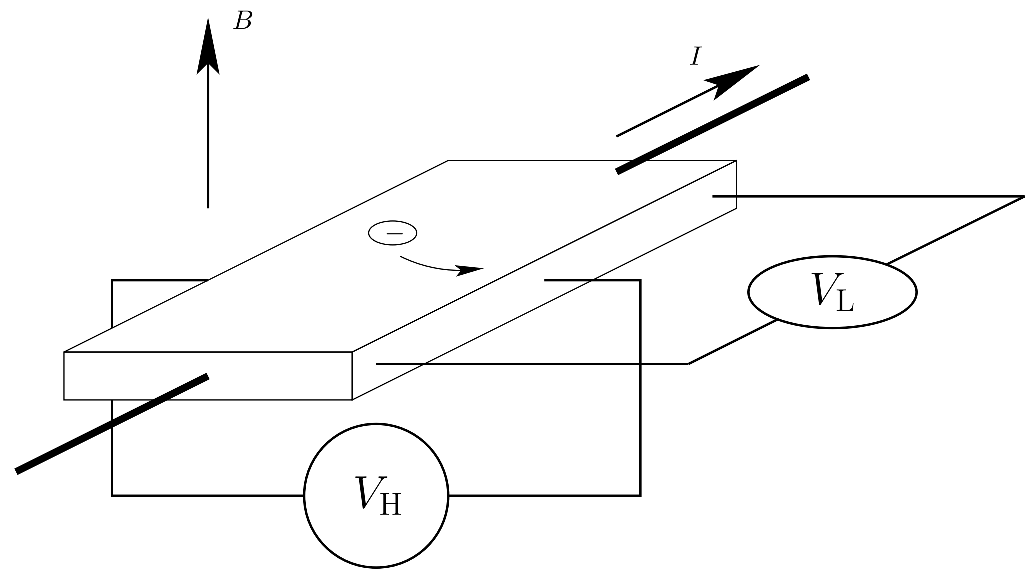
Charged particles that move in a magnetic field experience a ‘Lorentz force’, perpendicular to the field and direction they move in.

Example: earth magnetic field, giving rise to the northern light:



The classical Hall effect

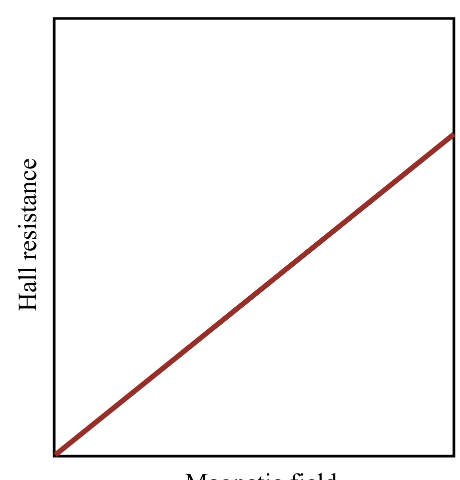
When there is a current through a thin strip, there is voltage drop along the current.
In a magnetic field, there is also a voltage perpendicular to the current, the Hall voltage.



The resistance is the ratio of the voltage to the current:

$$R_L = \frac{V_L}{I} \qquad R_H = \frac{V_H}{I}$$

The (classical) Hall resistance is proportional to the magnetic field:



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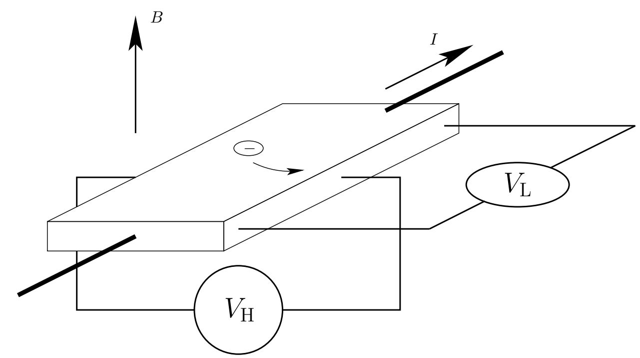

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$$\sigma_H = 1/R_H = \nu \frac{e^2}{h}$$

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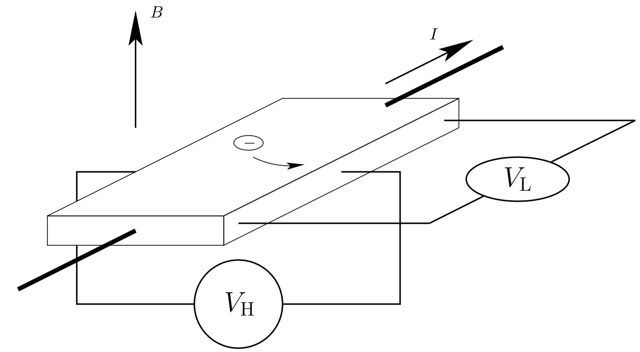

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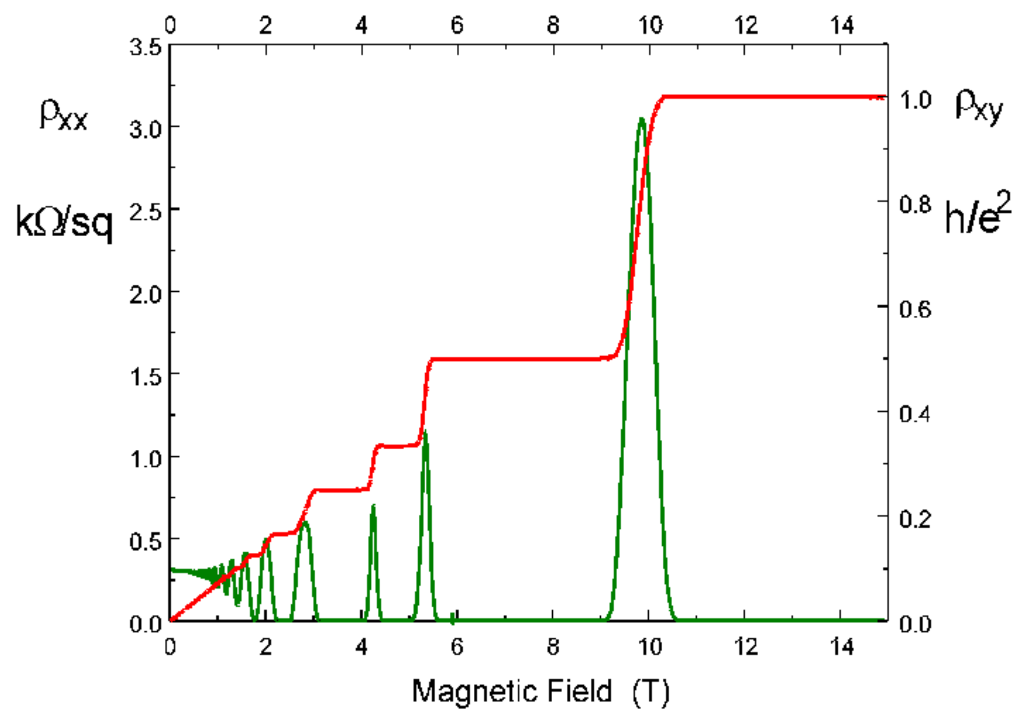
Red curve: Hall resistance
Green curve: longitudinal resistance



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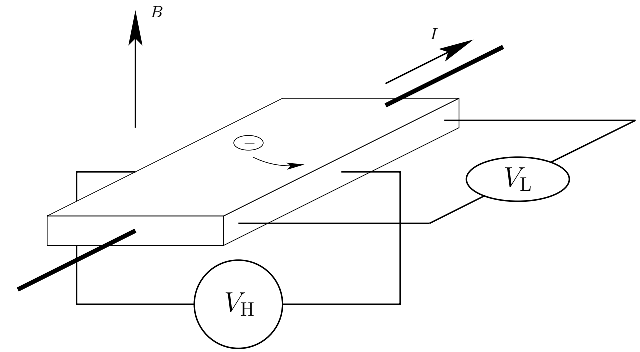

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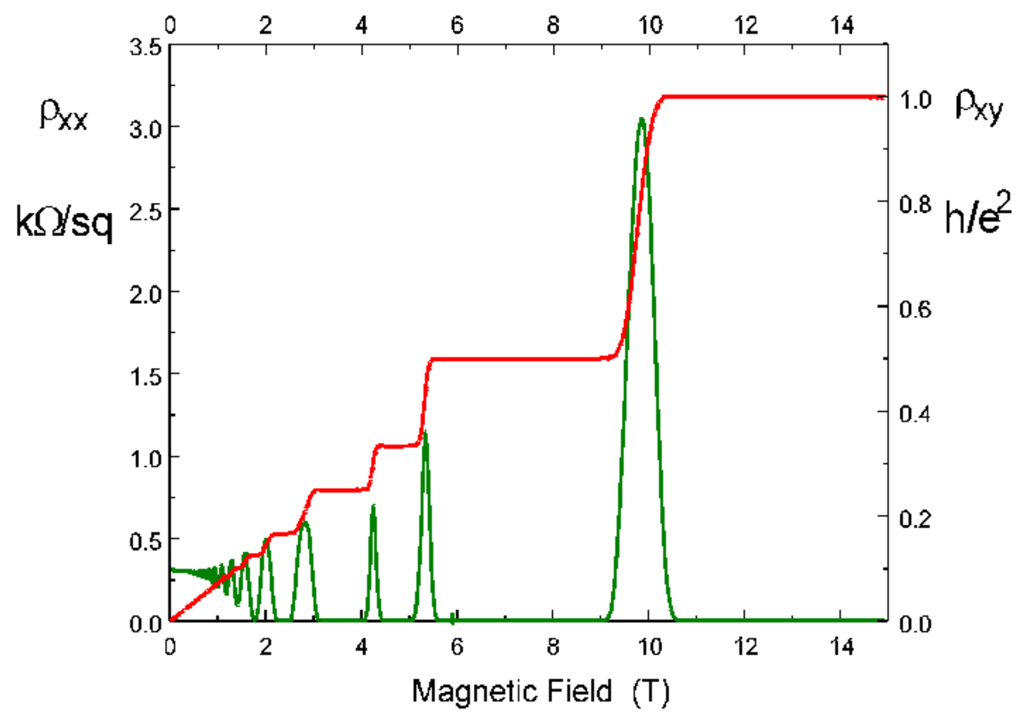
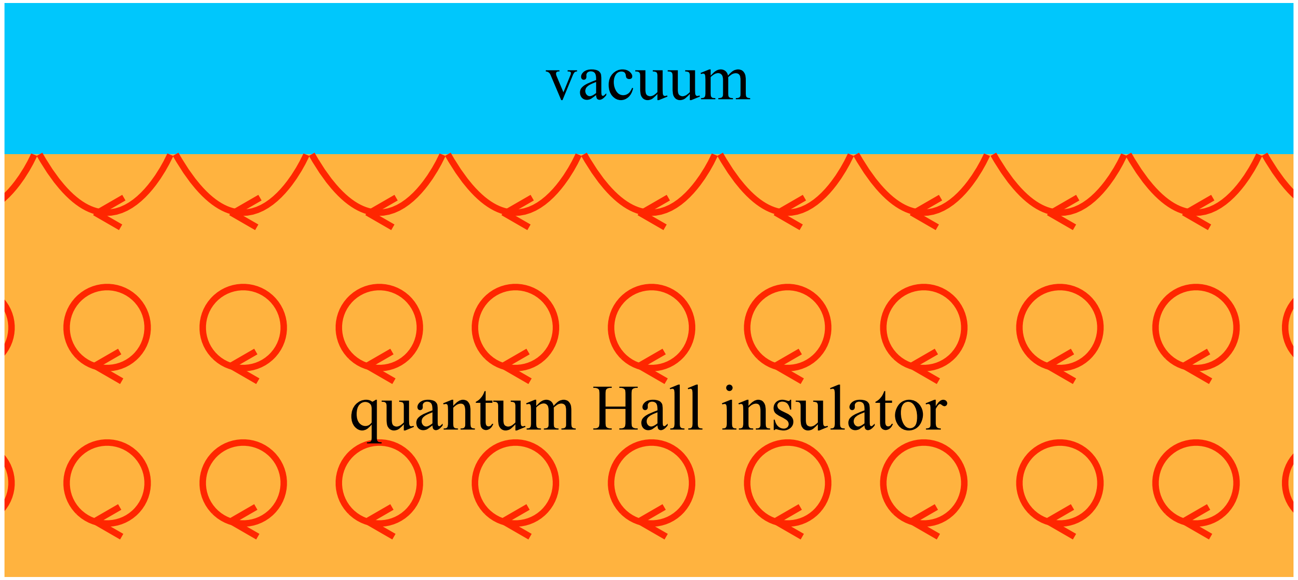
Hall resistance is a **topological invariant**



The quantum Hall effect

Inside the two dimensional droplet, the electrons are localized, and move in small circles (Larmor orbitals). However, electrons can move along the boundary, but only in one direction.



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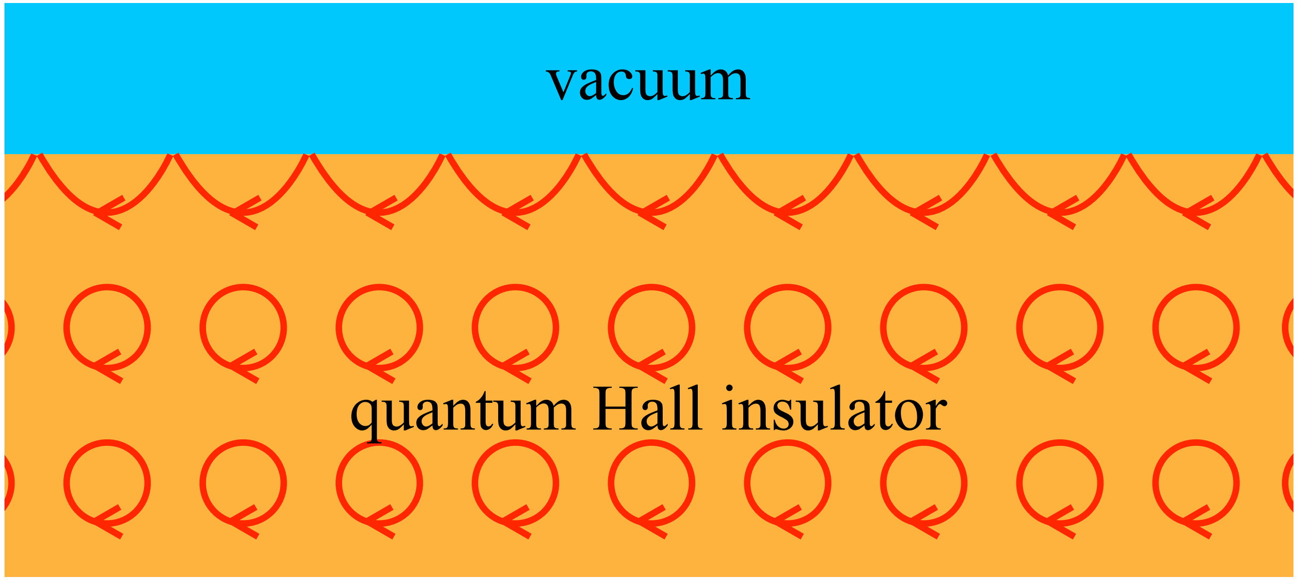

The quantum Hall liquid is an insulator, with a chiral edge mode!

The electrons on the edge can only move in one direction, which can not be changed, not even by dirt, disorder, etc. They can simply not turn back. This is why the quantization is so precise!

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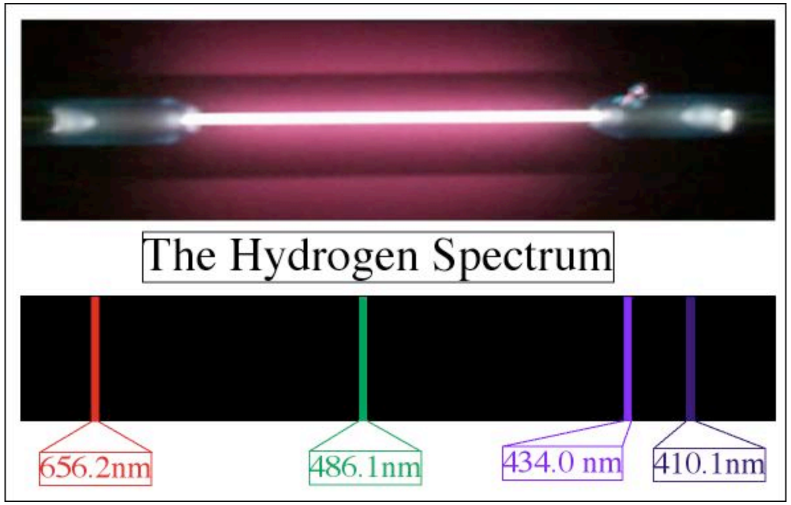
To explain this, we have to make a step back, and look at the difference between metals and insulators.

Energy levels

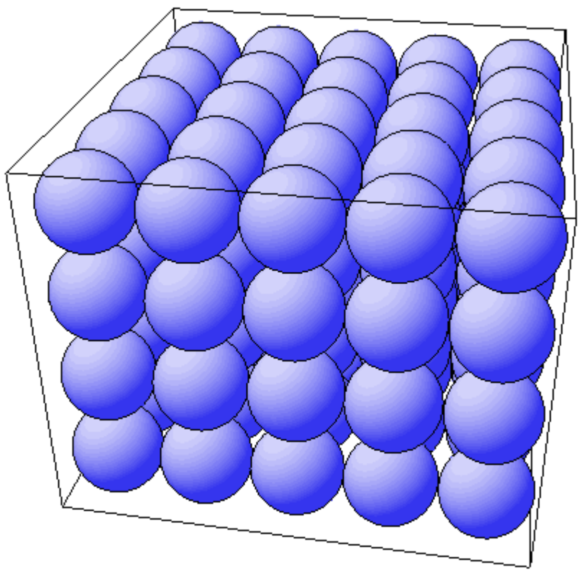
The energy levels of atoms, obtained by solving the Schrödinger equation, are discrete. For instance, for the hydrogen atom.

$$\mathcal{H}\Psi = E\Psi$$



Band theory

What happens if the electrons move in a periodic potential, such as in solids?



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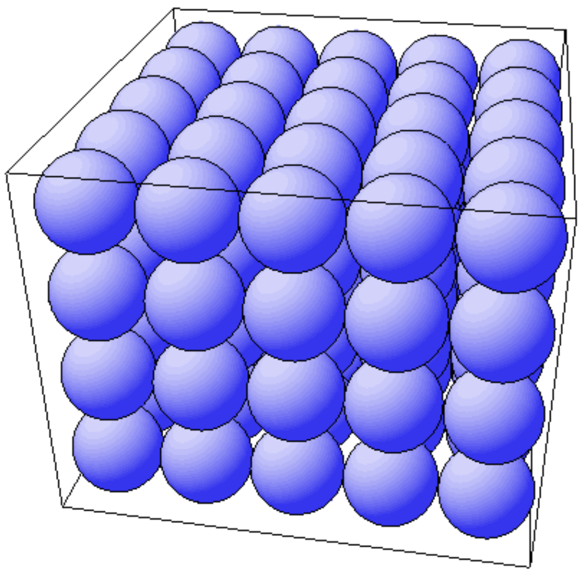


What happens if the electrons move in a periodic potential, such as in solids?

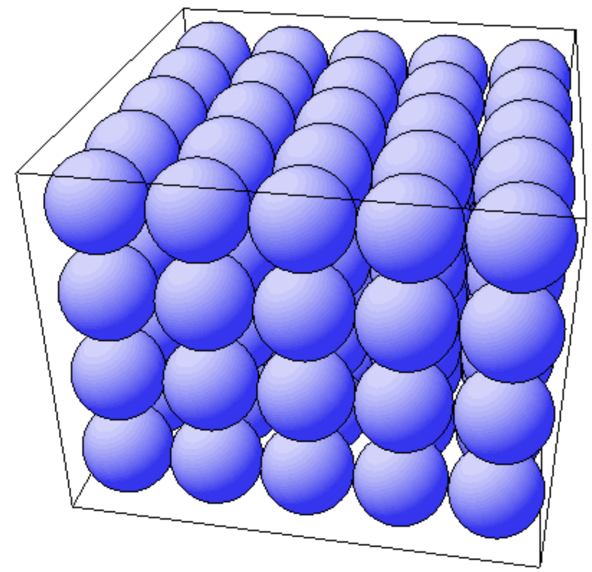
Bloch theorem says that the energy levels form continuous bands of states, that are periodic:

$$\Psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u(\vec{r})$$

'plane' wave periodic function

The energy of the states depends smoothly on the 'wave vector' k, forming a 'band', as opposed to discrete levels.

Band theory



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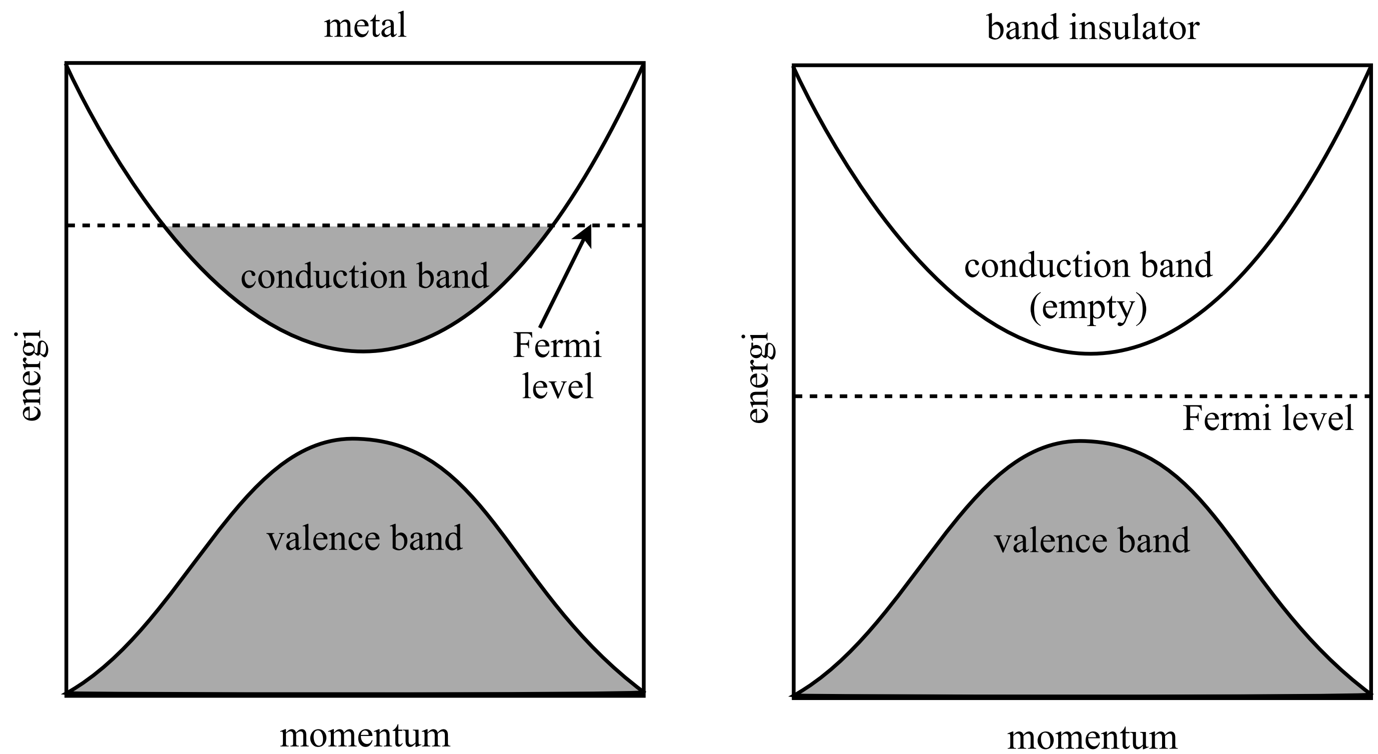
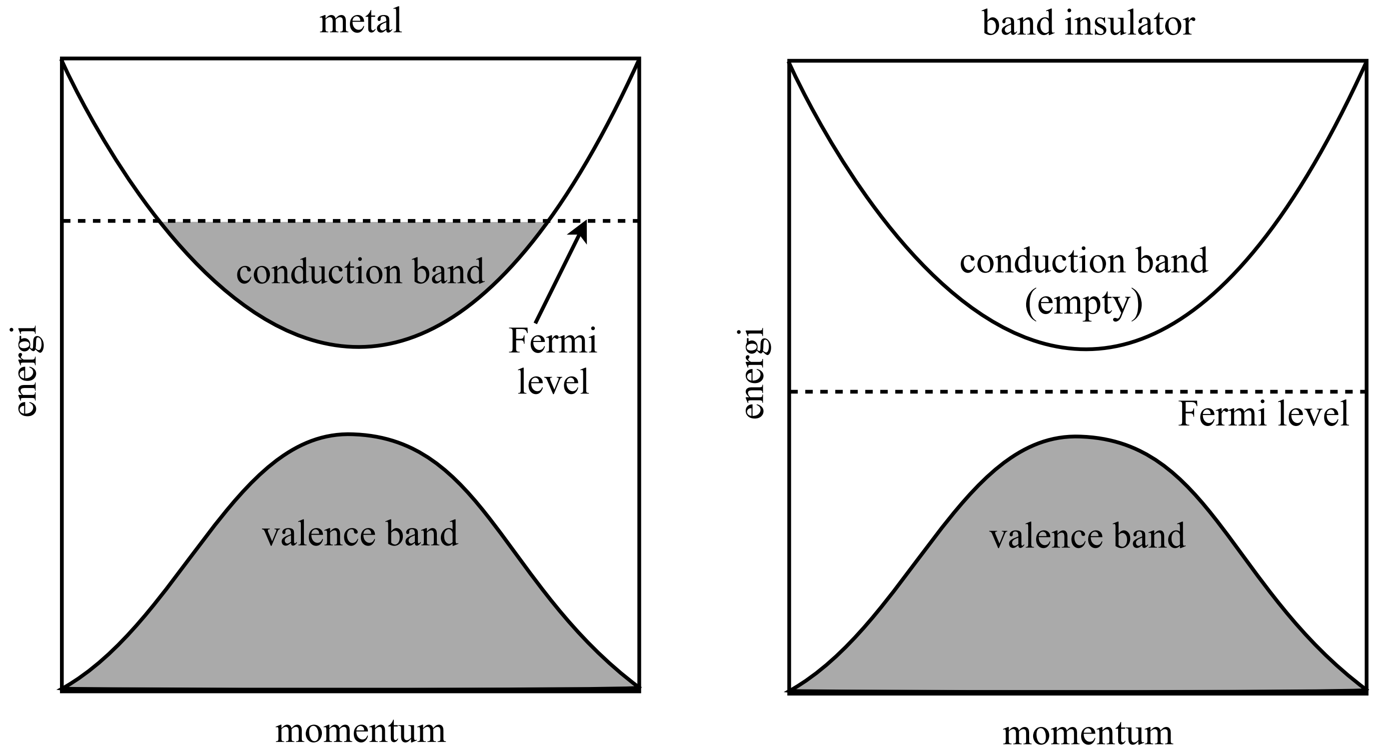
To what level the bands are filled, determines the properties of solids!

Metals v.s. insulators



Often, there is a gap between different bands. To which energy one has to fill each band (called the Fermi level), depends on the number of electrons that are not bound to an atom.

Metals v.s. insulators



In a metal, applying an electric fields give electrons slightly more energy, allowing it to conduct.

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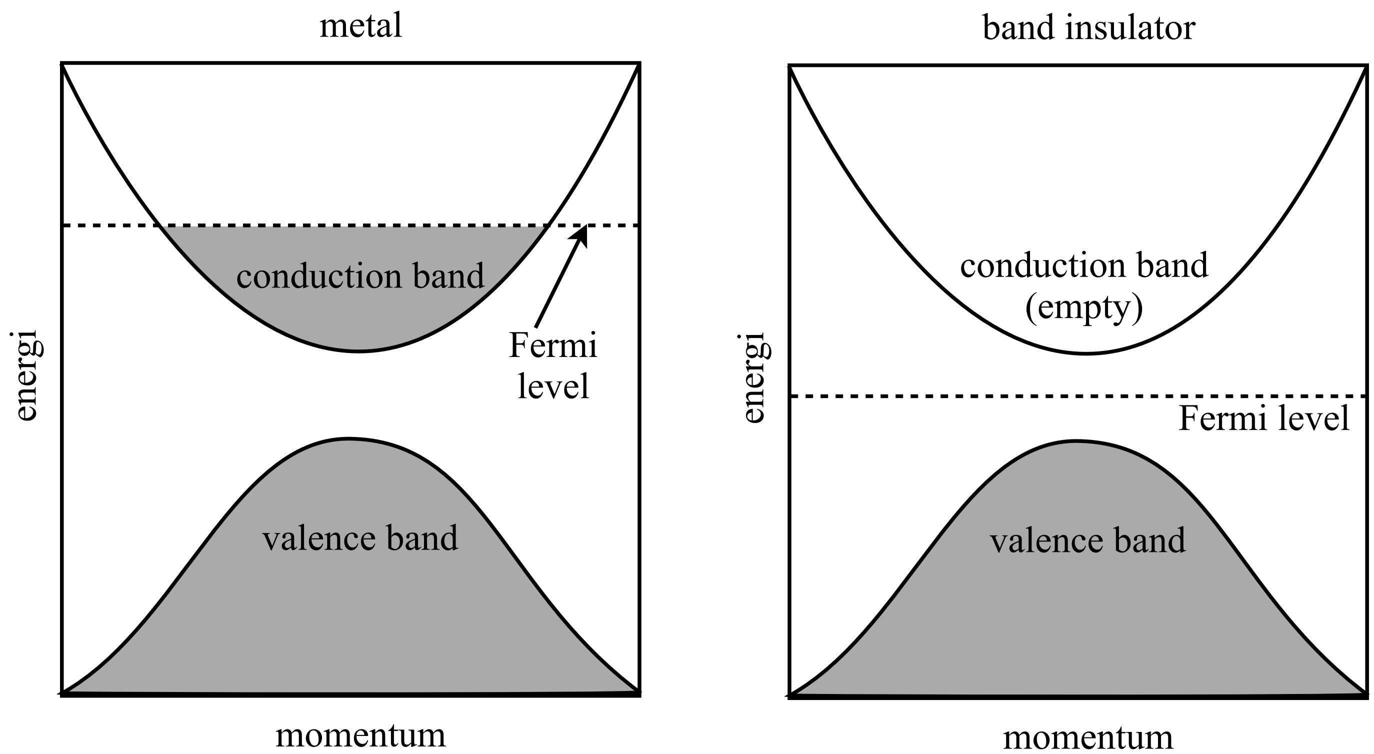

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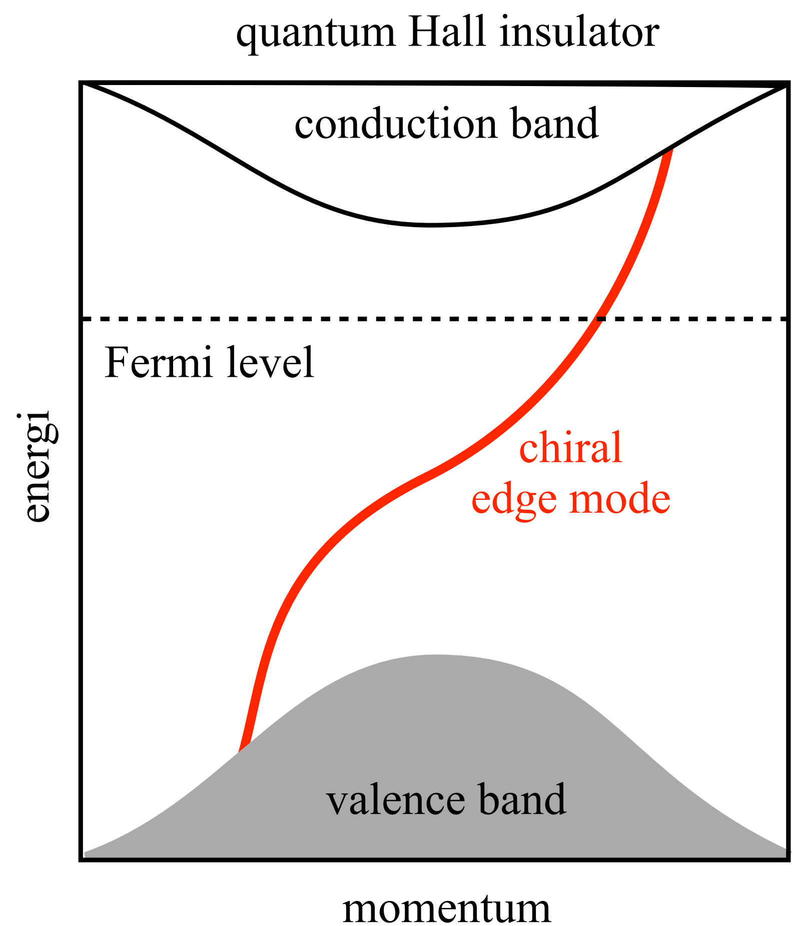
In an insulator, the only way to give an electron more energy, is to excite it over the gap, which costs a lot of energy. Therefore, insulators do not conduct for low electric fields!

Quantum Hall effect revisited

Using band theory, we can now also explain the situation in the quantum Hall effect.



There is a single energy level that connects the valence and conduction band. As long as the Fermi level is in the gap, there is a state that conducts!

The slope of the level gives the direction in which the electron moves, so we also find that this level is chiral.

Kane & Mele's model

Kane and Mele realized that topological phases can exist, even without a strong magnetic field.

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$$|\text{electron}>= \alpha|\uparrow> +\beta|\downarrow>$$

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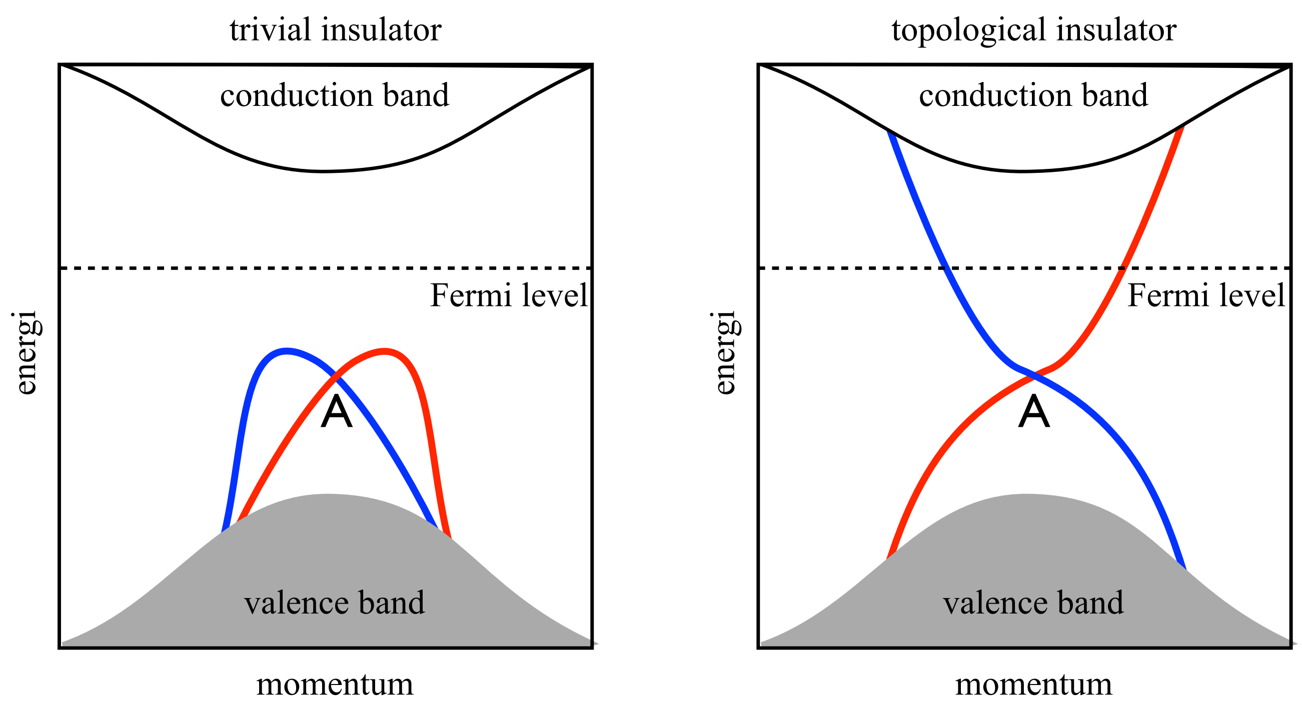
$$|\text{electron} >= \alpha| \uparrow> + \beta| \downarrow>$$

In the absence of magnetic fields, energy levels come in pairs, with exactly the same energy.

Magnetic fields can be **external**, or due to the motion of the electrons themselves, called **spin-orbit coupling**.

Kane & Mele's model

For certain values of the momentum, there are no magnetic fields, so the energy levels must come in pairs! Example: point A.



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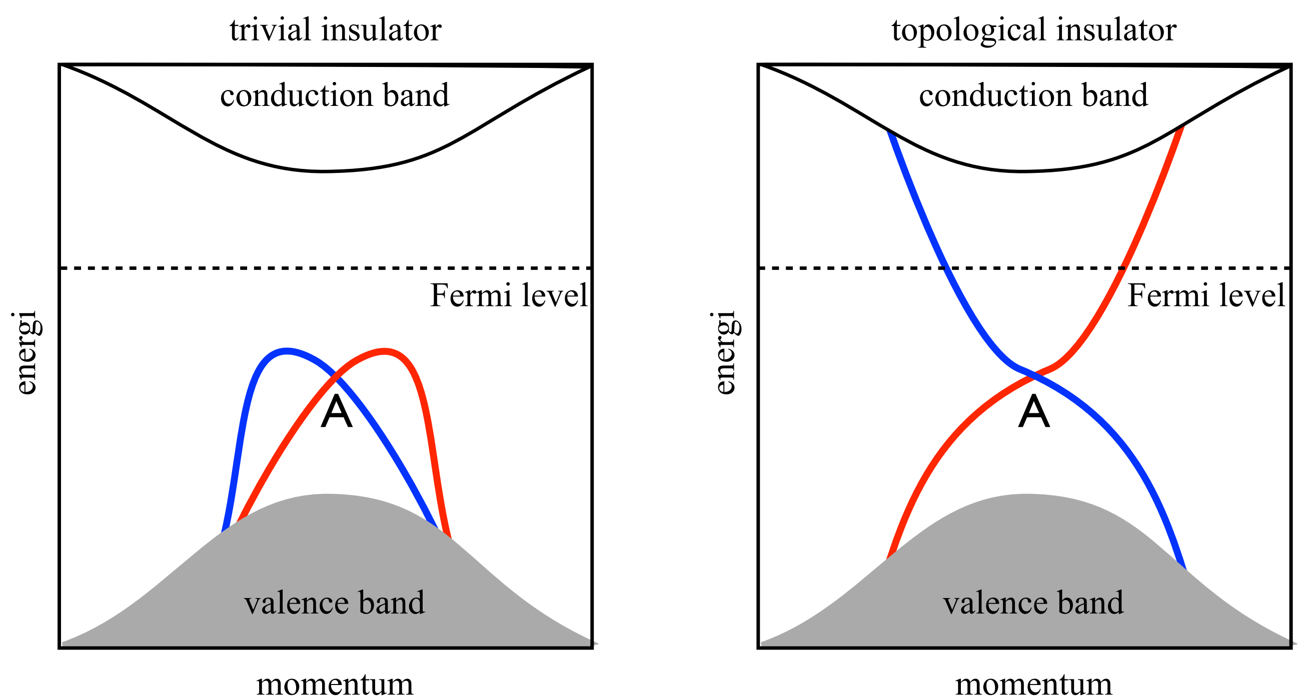


There are two, **topologically distinct** ways of drawing the diagram!

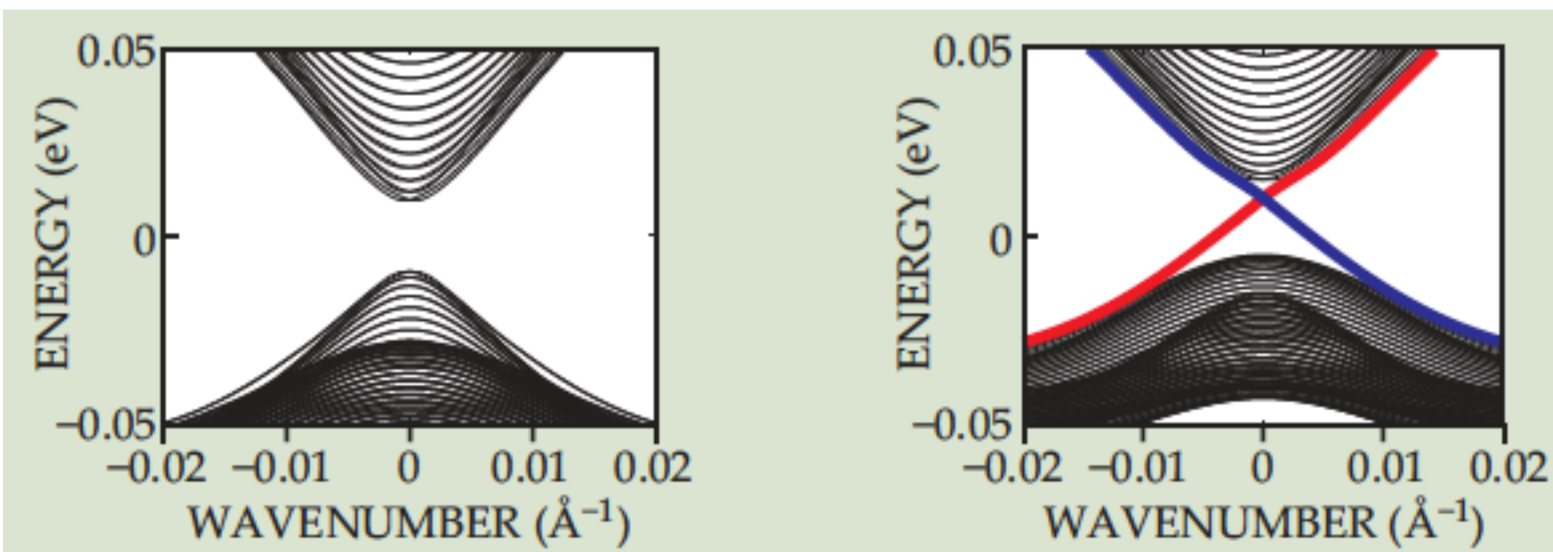
So there are two types of insulators, one with a pair of edge modes! These modes are chiral, and move in opposite direction.

Topological insulators: CdTe v.s. HgTe

Kane and Mele worked on graphene, which is always trivial.

Zhang & coworkers designed an experiment:
CdTe is trivial, while HgTe is topological

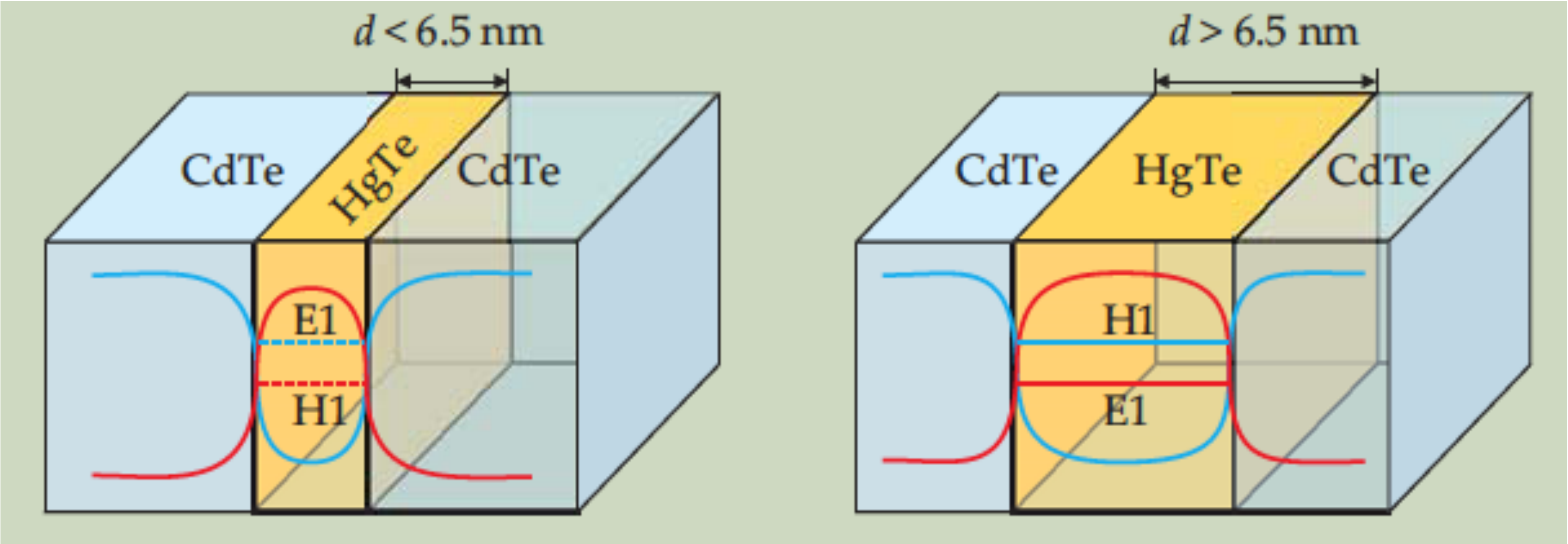
CdTe HgTe



Zhang et al., 2006 (Picture: Physics Today)

Topological insulators: CdTe v.s. HgTe

Zhang & coworkers designed an experiment:
Take a sandwich of CdTe-HgTe-CdTe, and vary the thickness!



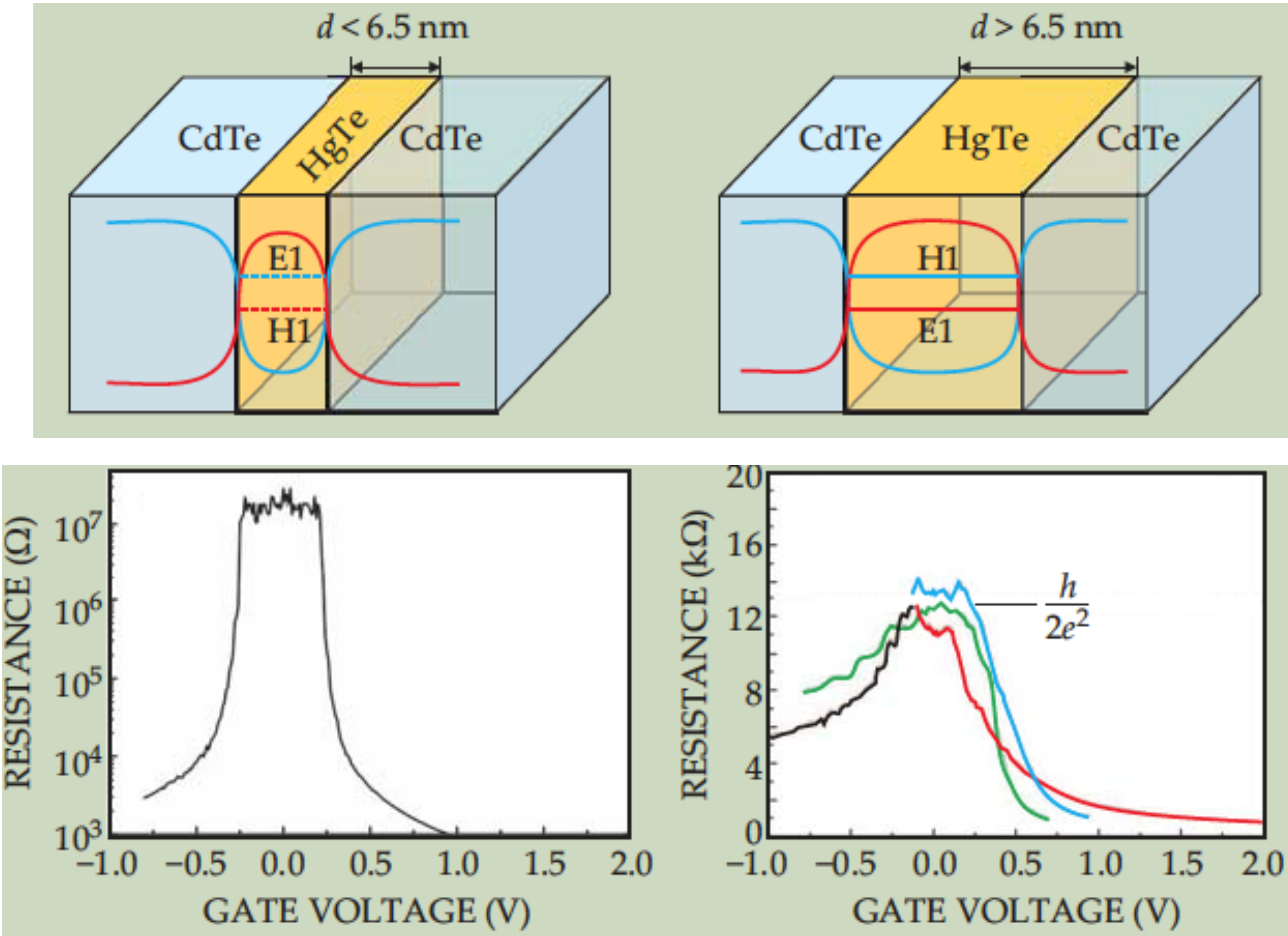
Trivial case

Topologial case

Zhang et al., 2006 (Picture: Physics Today)

Topological insulators: CdTe v.s. HgTe

Trivial case | **Topologial case**



Molenkamp et al., 2007 (Picture: Physics Today)

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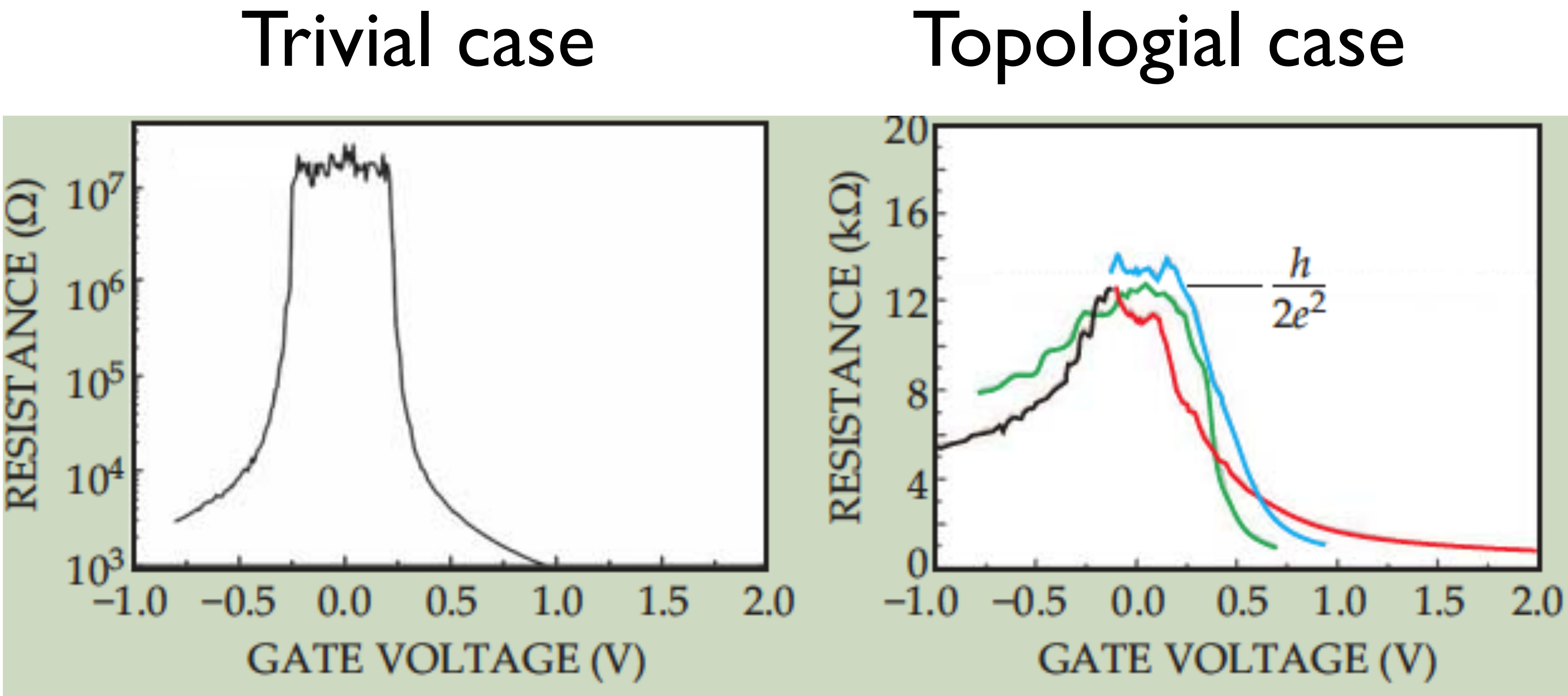

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The measurements show that the conductance is quantized in the topological case.
Other experiments show that the system indeed has chiral edge modes.

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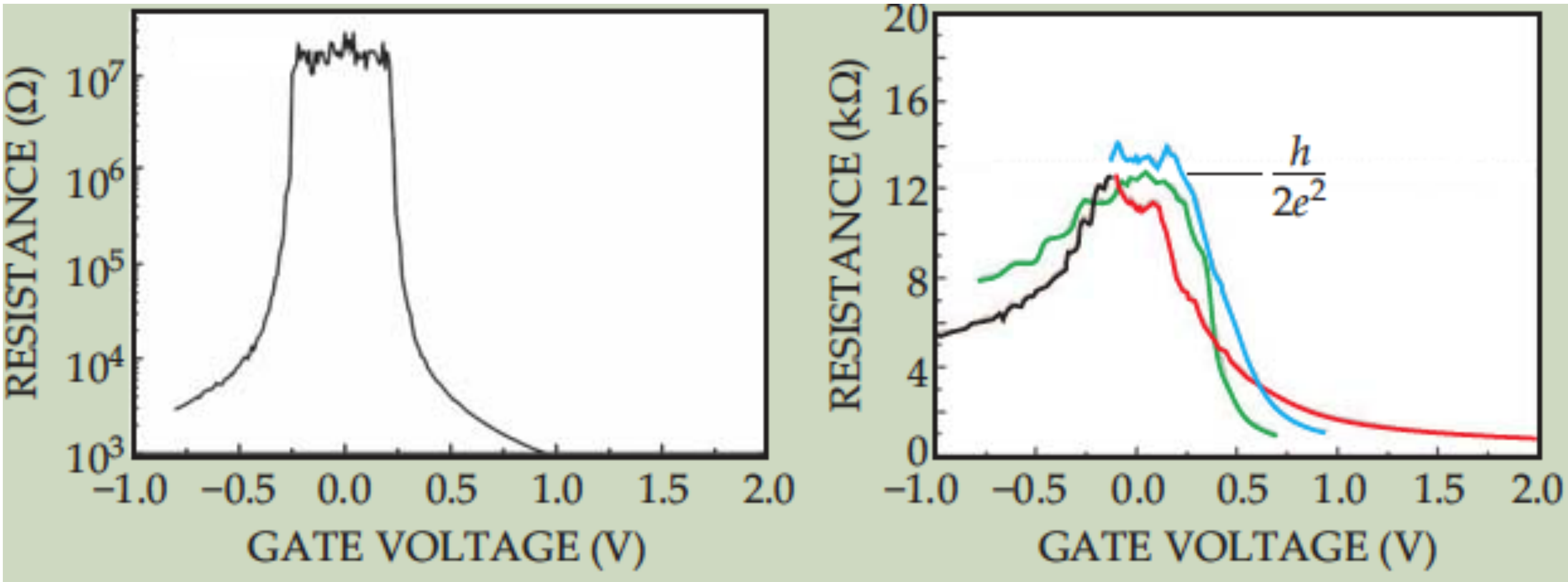
Trivial case | Topologial case



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Other experiments show that the system indeed has chiral edge modes.

Conclusion: this really is a topological insulator!
By now, many types are observed experimentally, in several groups!

Conclusions part I

In condensed matter, often experiments come (way) before the theory (superconductivity, quantum Hall effect for instance).

Topological insulators were predicted on theoretical grounds, and observed afterwards.

The topology in the problem allows for very precise predictions, that can be verified.

Topological insulators are extremely interesting from a fundamental perspective.

Can topological insulators be used for devices?