THE FUTURE OF BOSONIZATION

1. HISTORY
2. DIMENSIONS

   ZERO : X-RAY

   ONE : LUTTINGER, HUBBARD

   TWO : SQUARE FERMI SURFACE

   THREE : QED (QCD?)
3. WHY?

X-RAY THRESHOLD PROBLEM

SCHOTTE & SCHOTTE

TOMONAGA, LUTTINGER, LIEB & MATTIS

(INTERACTING ELECTRONS, SPINLESS, IN ONE SPACE DIMENSION)

$$H = \sum_k (k - k_F)\, a_{1k}^\dagger a_{1k} + (-k - k_F)\, a_{2k}^\dagger a_{2k}$$

$$+ \frac{g}{L} \sum_q \rho_1(q)\, \rho_2(-q)$$

a_{1k}, a_{2k} — FERMIONS

$[a_{1k}, a_{1k'}^\dagger]_+ = \delta_{k,k'}$

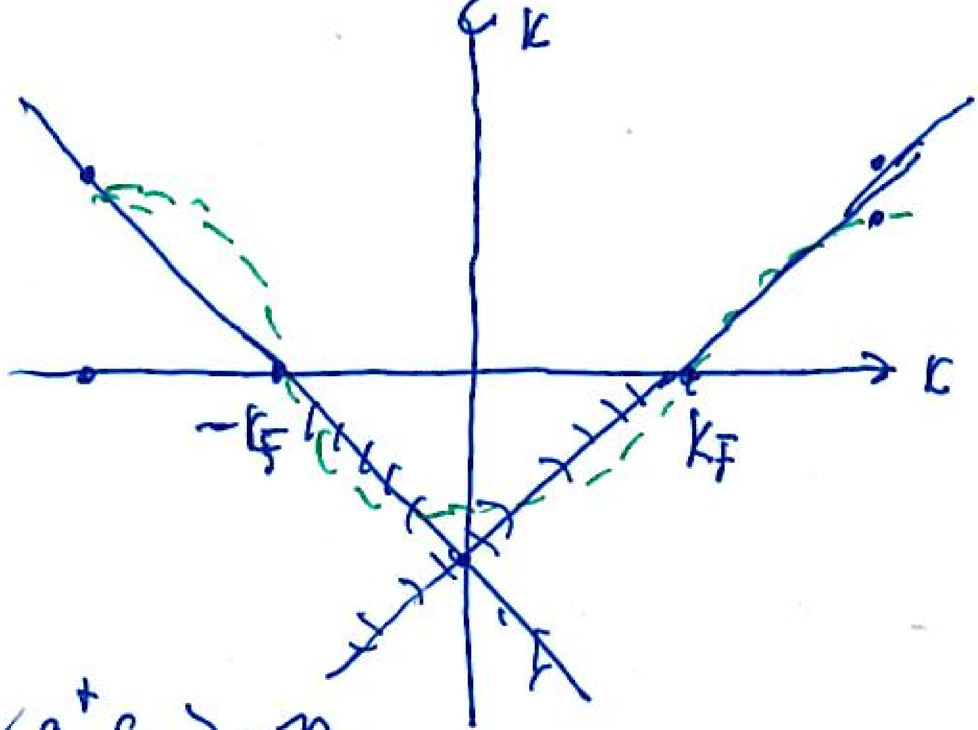
$\rho_1(q) = \sum_k a_{1k+q}^\dagger a_{1k}$ etc.

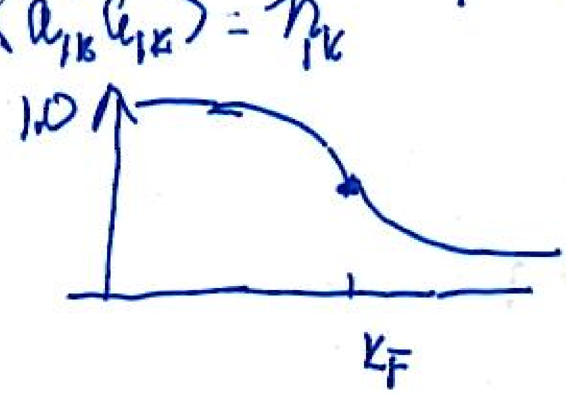
$k = \frac{2\pi n}{L}$ (n integer)

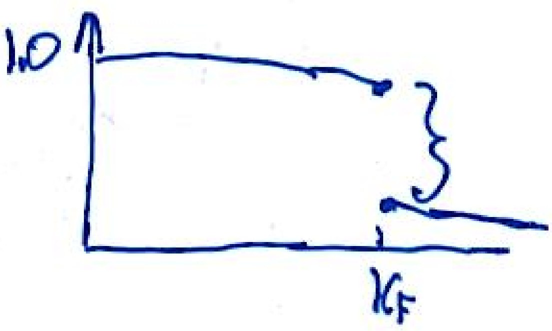
SURPRISE: SCHWINGER ANOMALY

$[\rho_1(q), \rho_1(q')] = \frac{q' L}{2\pi} \delta_{q,-q'}$

BOSONS







LUTTINGER MODEL
(AT ZERO TEMPERATURE)

LANDAU FERMI LIQUID

$$\rho_1(q) = \sum_k a^\dagger_{1k+q}\, a_{1k}$$

BOSON FERMIONS

THE OTHER WAY AROUND?

$$\psi_1^\dagger(x) = \frac{1}{\sqrt{L}} \sum_k a^\dagger_{1k}\, e^{-ikx}$$

AND $$= Z\, e^{2ik_F x + \phi_1(x)}$$

$$\phi_1(x) = \frac{2\pi}{L} \sum_{k\neq 0} \frac{\rho_1(k)}{k}\, e^{-ikx}$$

I. PESCHEL & A. L.

COLEMAN, MANDELSTAM, MATTIS

CONNECTIONS

STATISTICAL MECHANICS

1. 8-VERTEX MODEL (BAXTER)

FIELD THEORY

2. THIRRING MODEL

3. MASSIVE THIRRING MODEL

SOLID STATE

4. X-Y-Z SPIN 1/2 CHAIN

BOOKS

TSVELIK, NERSESYAN, STONE, MATTIS

1D HUBBARD MODEL (SPIN)

$$H = \sum_{k,\sigma} (-t \cos ka)\, c^{\dagger}_{k\sigma} c_{k\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

(SAME SITE REPULSION)

BOSONIZATION GIVES NEW PHYSICS:

SPIN - CHARGE SEPARATION
(PESCHEL, EMERY, A.L.)

$$H \to H_{SPIN} + H_{CHARGE}$$

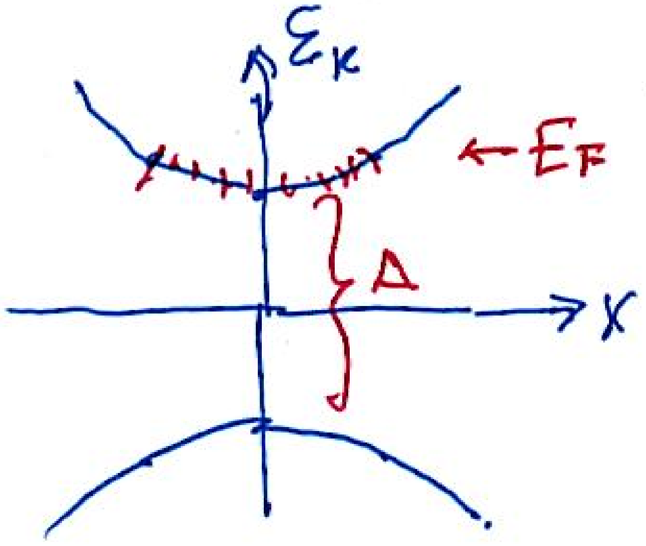
(NOT FERMI LIQUID SINCE A QUASI-PARTICLE EXCITATION CARRIES BOTH.)

NEED SUSCEPTIBILITIES!

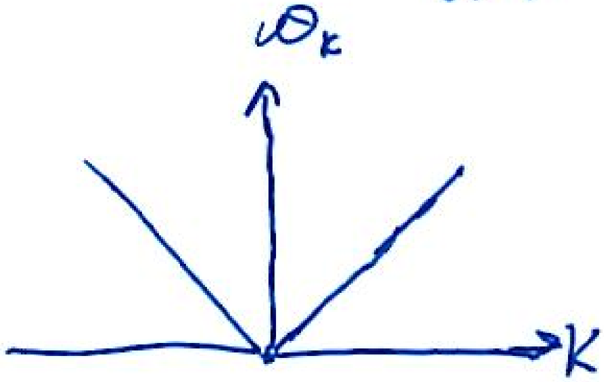
LUTTINGER: $\chi_{SC}(T) = \chi_0 \left(\frac{T}{T_0}\right)^{P}$

$P > 0$ FOR REPULSION

1D HUBBARD



CHARGE



SPIN

$E_F = 0$ AT HALF FILLING

$E_F > \frac{\Delta}{2}$ <u>TWO</u> PROCESSES

1. EXCITATIONS AT E_F (CFT)

$$\chi_{SC} = \chi_0 \left(\frac{T}{T_0}\right)^P \quad P \gg 0$$

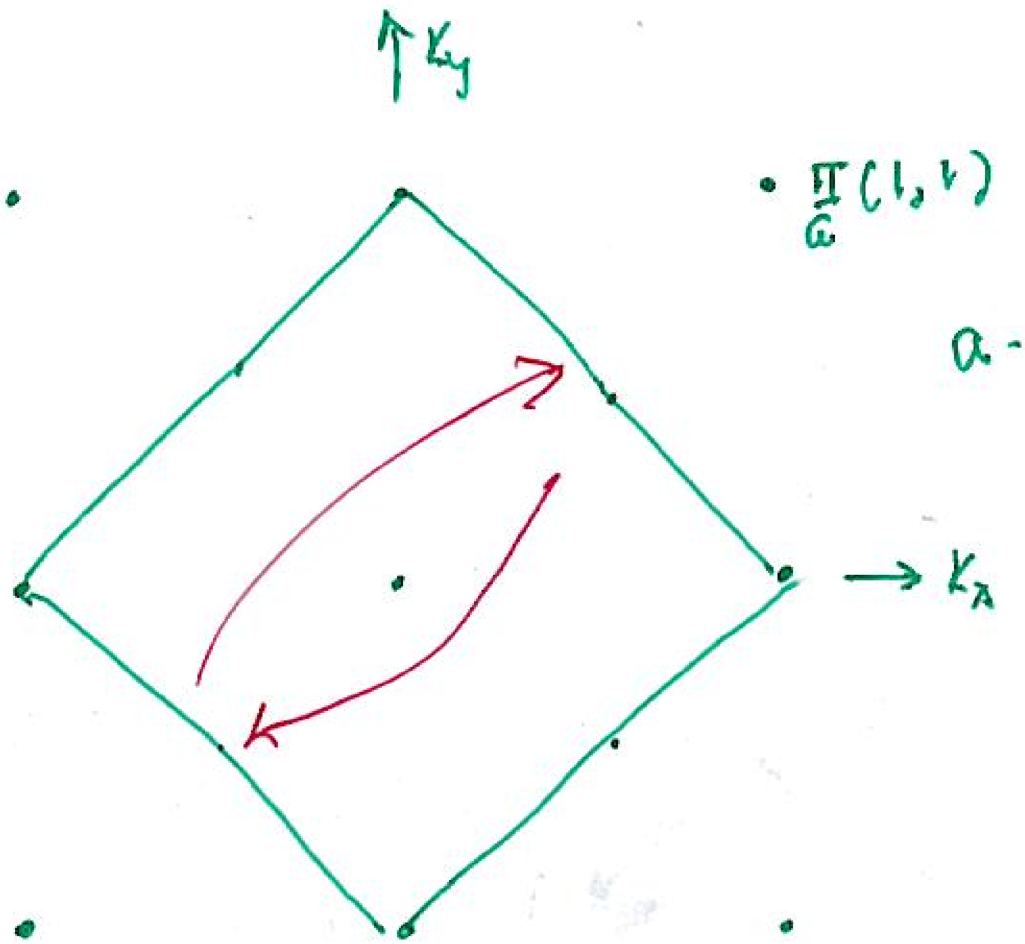
$U > 0$

2. PAIR EXCHANGE?

$$\frac{V^2}{(-\Delta)} \quad ?$$

CAN THIS DOMINATE?

2D HUBBARD



a - lattice spacing

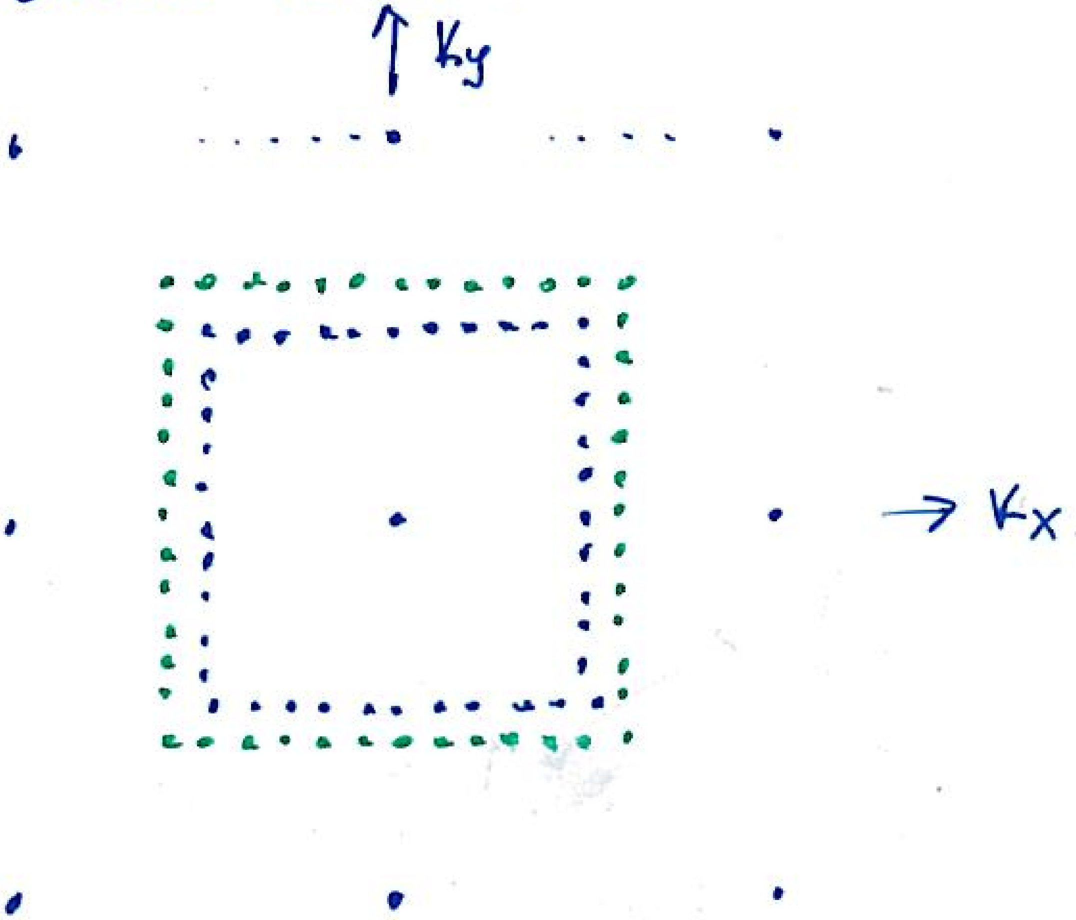
SQUARE FERMI SURFACE, AT HALF FILLING
(ONE ELECTRON PER SITE)
NEAREST NEIGHBOR HOPPING

$$H = \sum_{\substack{\langle ij \rangle \\ \sigma}} t\, a^{+}_{i,\sigma} a_{j,\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

(n.n. sites) (SITES)

SUGGESTED NEW MODEL

ENERGY SQUARES



$$E(k_x, k_y) = v\, k_x \qquad (-k_x < k_y < k_x)$$

ALL STATES ON A SQUARE ARE DEGENERATE.

e.g.

$$a_\ell(k_x) = \sum_{k_y} e^{i\ell k_y}\, a(k_x, k_y)$$

ℓ – "ANGULAR" MOMENTUM

AT HALF FILLING → HUBBARD MODEL

LUTTINGER ↔ HUBBARD ?

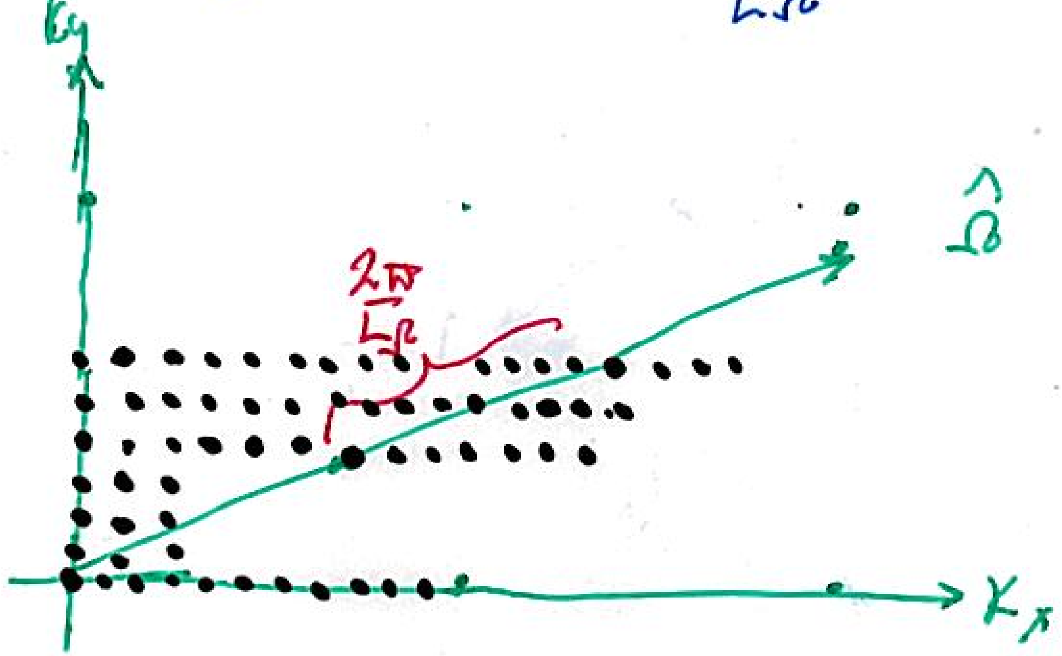
WHY THIS SQUARE ENERGY MODEL?

1. CAPTURES PHYSICS OF 2D HUBBARD
2. INTEGRABLE → EXACT
3. NEED EXACT TO MAKE BELIEVABLE PREDICTIONS
4. WANT TO FIND CONDITIONS FOR ROOM TEMPERATURE SUPERCONDUCTORS.

IN 3D QED [10]

$$\vec{A}(\vec{x}) = \frac{1}{L^{3/2}} \sum_{k,\lambda} \frac{\left(b^{+}_{\vec{k},\lambda} + b_{-\vec{k},\lambda}\right)}{\sqrt{2|\vec{k}|}} e^{-i\vec{k}\cdot\vec{x}} \hat{\epsilon}_{\lambda}$$

$$\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z) = \frac{2\pi}{L_\Omega} n \hat{\Omega}$$



e.g. 2D

EQUALLY SPACED POINTS

(SCHOTTE & A.L.) BOSON ⟷ FERMION DUALITY

CONNECTION TO NUMBER THEORY

$\vec{A}(x) \rightarrow$ FERMIONS $b^{+}_{\vec{k}} \sim \sum_{m} a^{+}_{n+m,\Omega} a_{m,\Omega}$

$\vec{J}\cdot\vec{A}$ HAS <u>ONLY</u> BACKSCATTERING.

INTEGRABLE.

QED EIGENSTATES IN 3+1

$$H = \sum_{\vec{k}} \bar{\psi}_{\vec{k}} (\vec{k}\cdot\vec{\alpha} + \beta m)\psi_{\vec{k}} + e^{2}\sum_{\vec{k}} \vec{J}(k)\cdot\vec{A}(k)$$

$$+ \sum_{\vec{k},\lambda} \omega_k b^{+}_{k\lambda} b_{k\lambda}$$

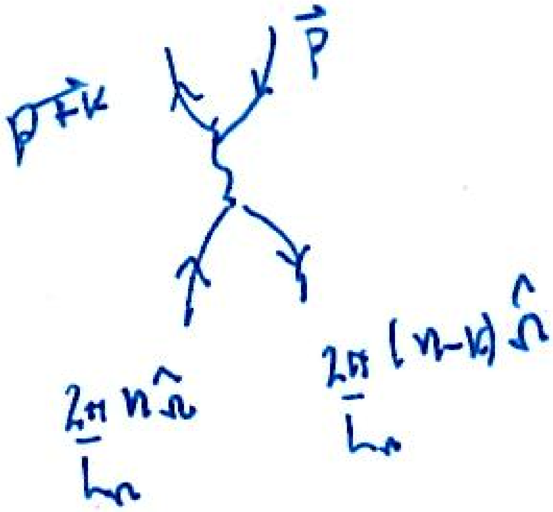
$$\vec{J}(k) = \sum_{\vec{p}} \bar{\psi}_{p+k}\,\vec{\gamma}\,\psi_{\vec{p}}$$

$$\vec{A}(\vec{k}) = \frac{1}{L^{3/2}}\sum_{\lambda} \frac{\hat{e}_\lambda (b^{+}_{k,\lambda} + b_{-\vec{k},\lambda})}{\sqrt{2k}}$$

$$\rightarrow \sum_{\lambda\, n} \hat{c}_\lambda \frac{C^{+}_{\Omega}(n+k)\,C_{\Omega}(n)}{n}$$

$$\vec{k} = \frac{2\pi k}{L_\Omega}\hat{\Omega}$$

HAS ONLY BACKSCATTERING



SUMMARY

BOSONS $\leftrightarrow$ FERMIONS

1D: NEED USER FRIENDLY EIGENFUNCTIONS IN ORDER TO CALCULATE SUSCEPTIBILITIES

2D: THE "SQUARE" MODEL, TO UNDERSTAND 2D HUBBARD MODEL. ARE THERE BIG SUPERCONDUCTING FLUCTUATIONS?

3D: QED AS A PURELY FERMION BACKSCATTER PROBLEM. QCD?

SUPERSYMMETRY $\leftrightarrow$ DUALITY

$\neq$

WHAT ARE THE GENERATORS? THE GROUP?

WHERE IS MS. KING?

CONNECTION TO NUMBERTHEORY

MÖBIUS FUNCTION

$$\sum_{m=1} \mu(m)\, f(mn) = \delta_{n,1}\, f(1)$$

m,n integers $\qquad \mu(m) = \begin{matrix} +1 \\ -1 \\ 0 \end{matrix}$

$$\sum_{\Omega} e^{\frac{2\pi i\, \hat{\Omega}\cdot\vec{x}}{L_n}} = \,?$$

$$\sum_{\vec{k}} = \sum_{\Omega} \sum_{n}$$

$$\left(\vec{k} = \frac{2\pi n\, \hat{\Omega}}{L_n} \right)$$

$$\sum_{\Omega} e^{\frac{2\pi i\, \hat{\Omega}\cdot\vec{x}}{L_n}} = \sum_{m,n,\Omega}$$

$$\sum_{m,n,\Omega} e^{i m \vec{k}\cdot\vec{x}} \mu(m) = \sum_{m,n,\Omega} e^{\frac{2\pi i\, mn\, \hat{\Omega}\cdot\vec{x}}{L_n}} \mu(m)$$

$$= \sum_{\Omega} e^{\frac{2\pi i\, \hat{\Omega}\cdot\vec{x}}{L_n}}$$

$$= \sum_{\vec{k},m} \mu(m)\, e^{i m \vec{k}\cdot\vec{x}}$$