In search of topological states with fractionalized excitations.

Eun-Ah Kim

Cornell University

Buzz words

Topology

Quantum Hall

Causality

Edge State

non-Abelian

chiral superconductor

Event space

Point contact

Covariance

Chern-Simons theory

spinfluid density

Braiding

Wilson line

Topology for everyone

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Why topologial states in CMP?

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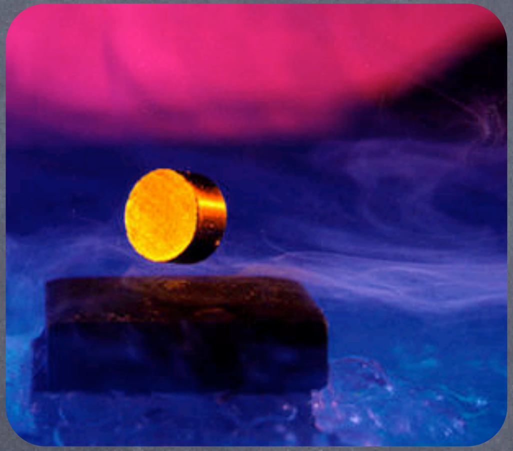

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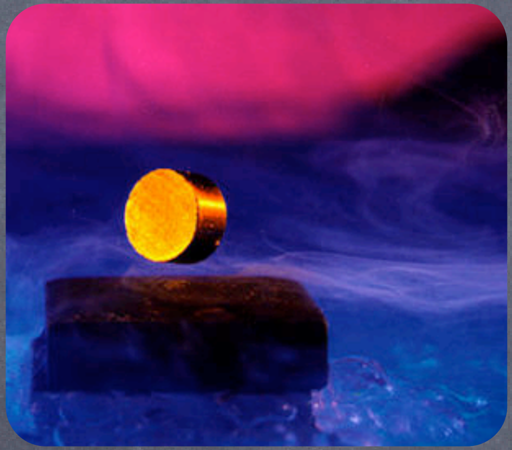

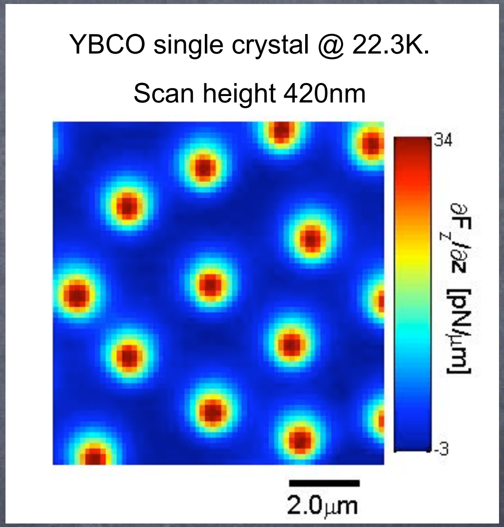

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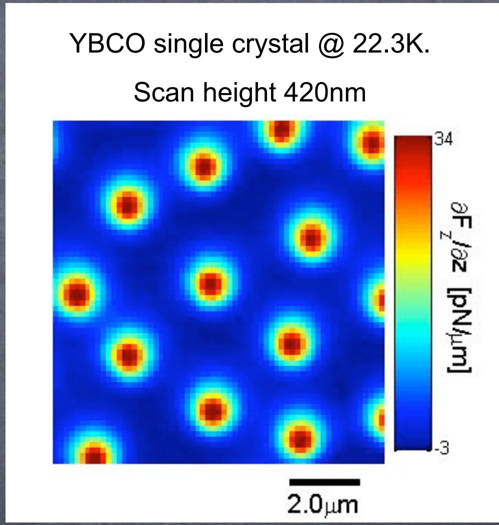

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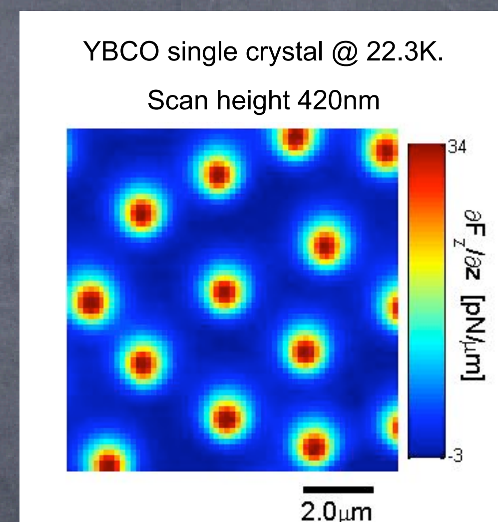

- Topological order a new paradigm
  for many body physics.

In search of topological states with fractionalized excitations.

- Topological order and fractionalization
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- Stability of 1/2-qv's in SrRuO
- Summary and outlook

Ground state degeneracy

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Conventional order

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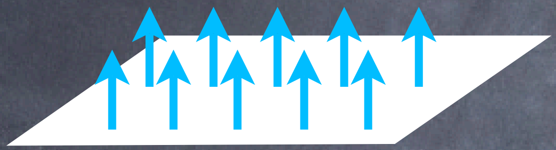

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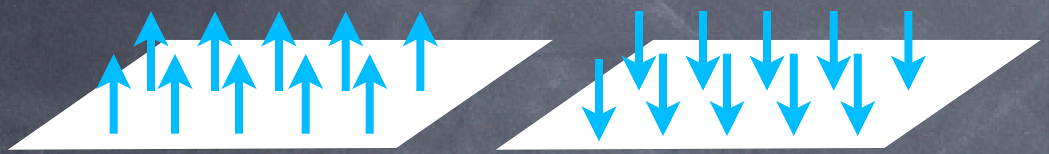

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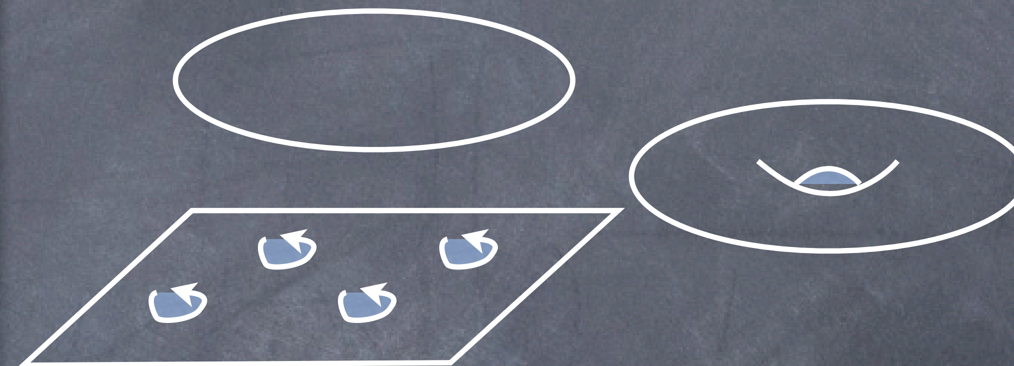

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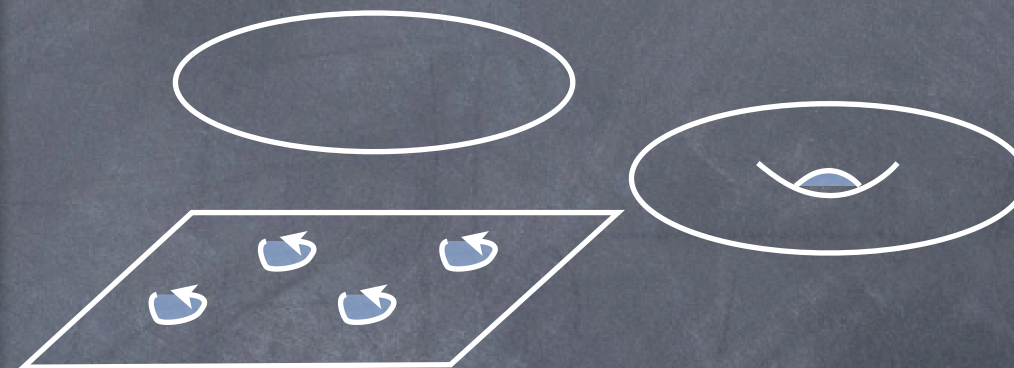

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Where to look?

p-wave Feshbach resonance

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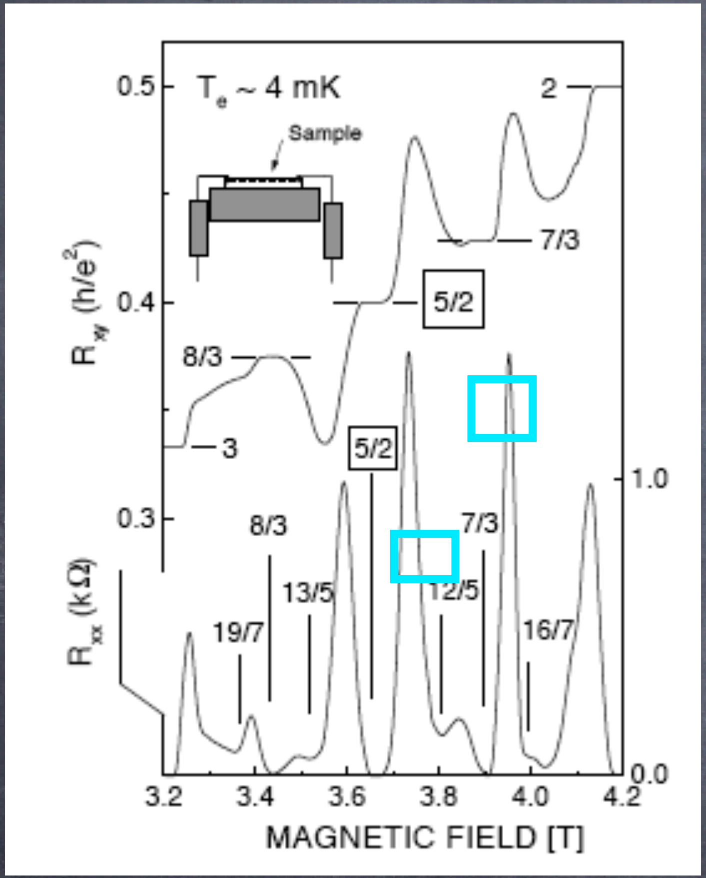
- FQH states



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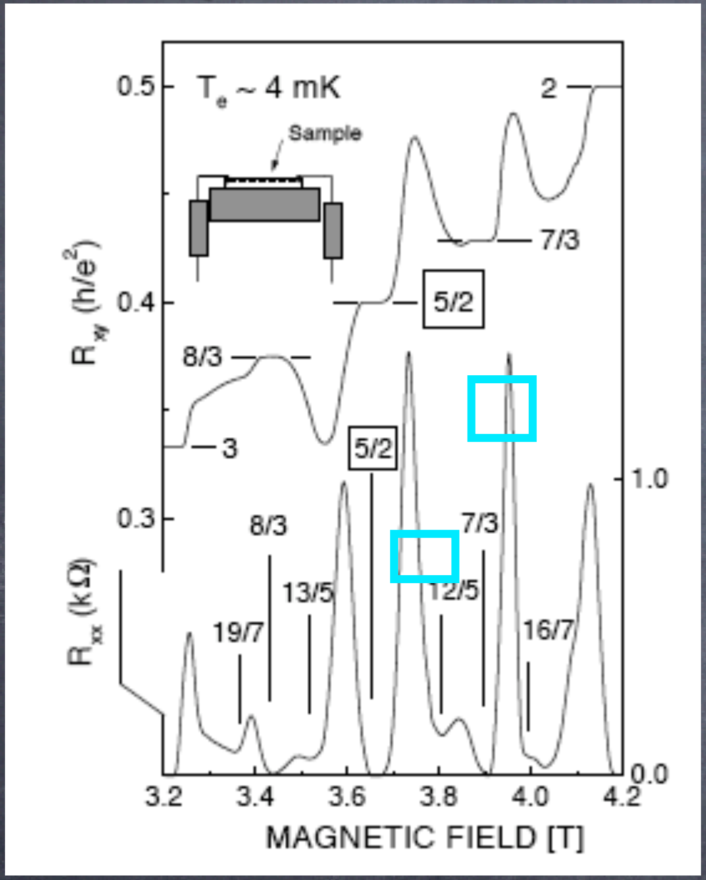

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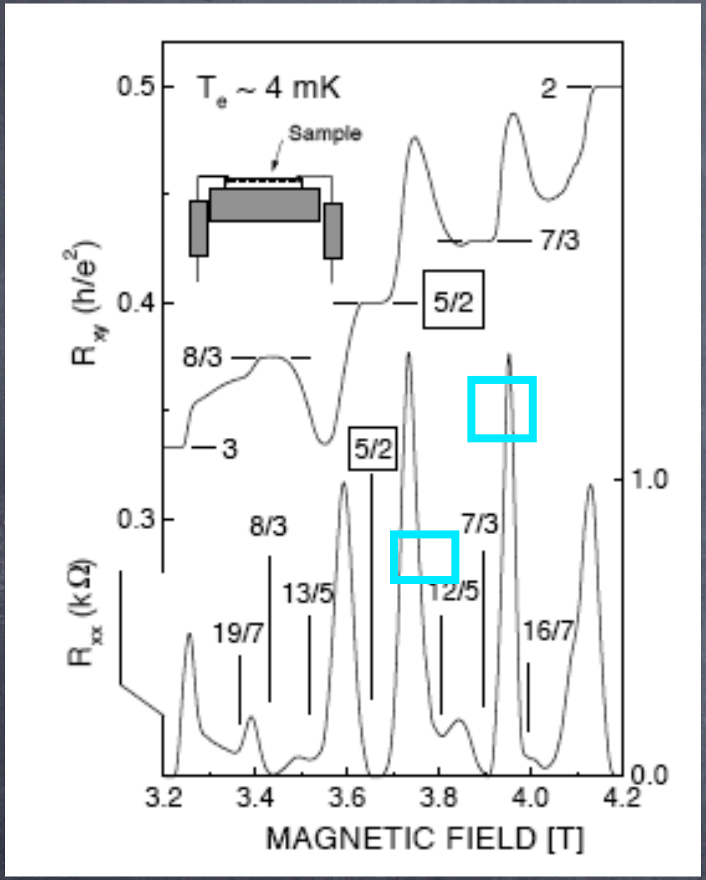

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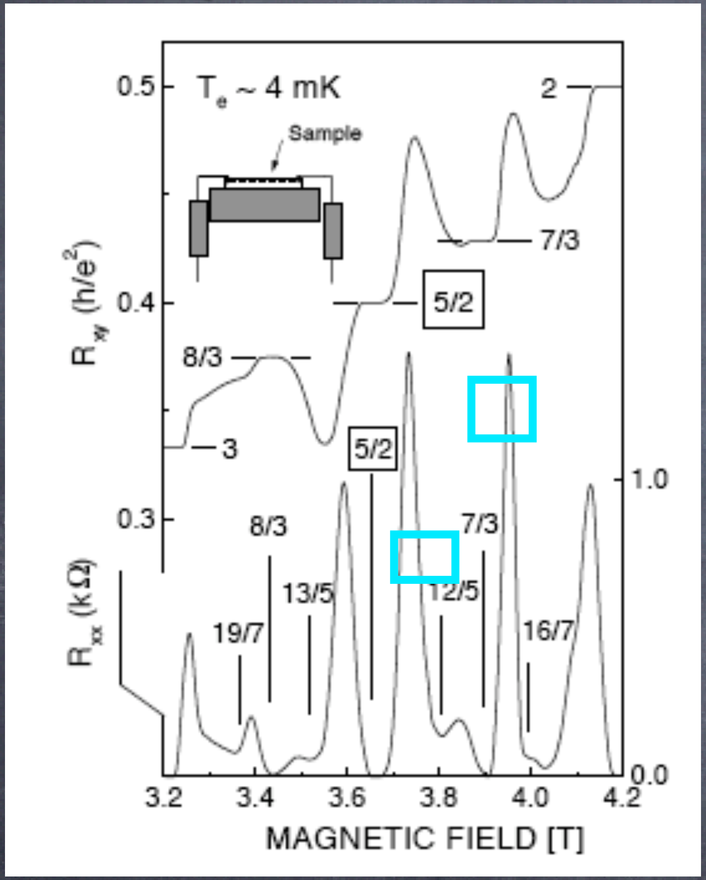

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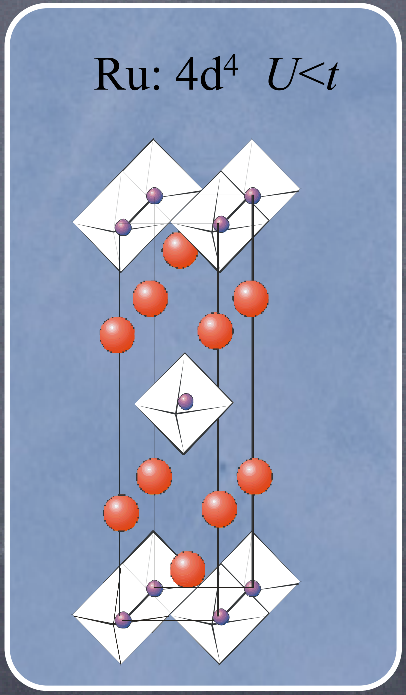

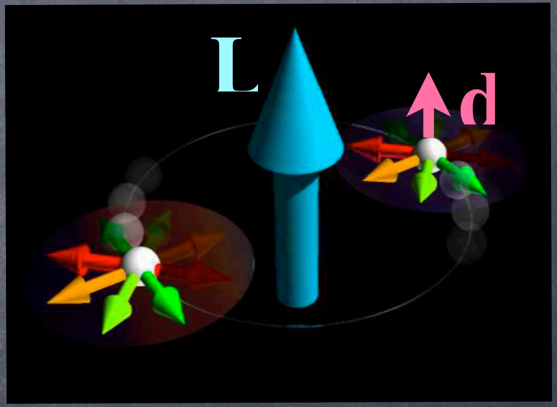

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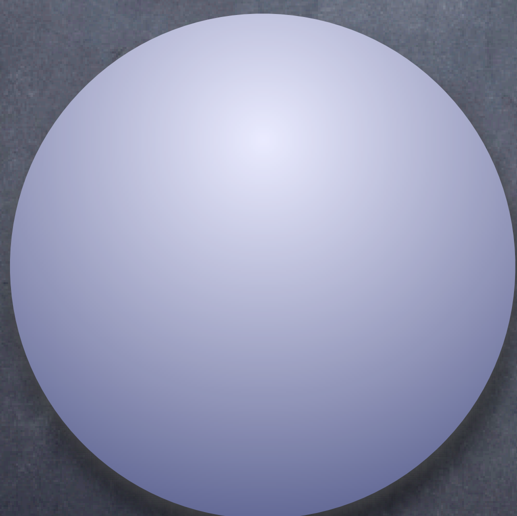

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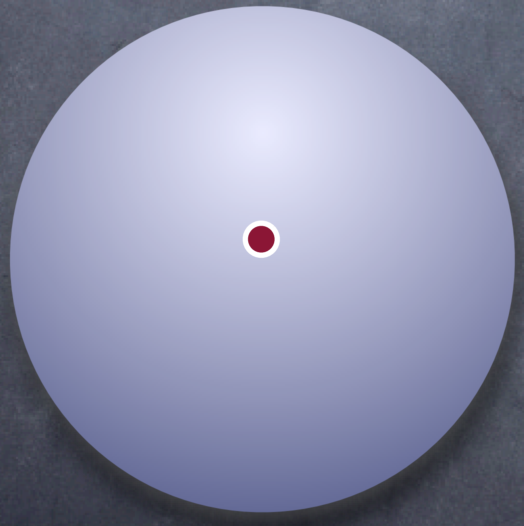

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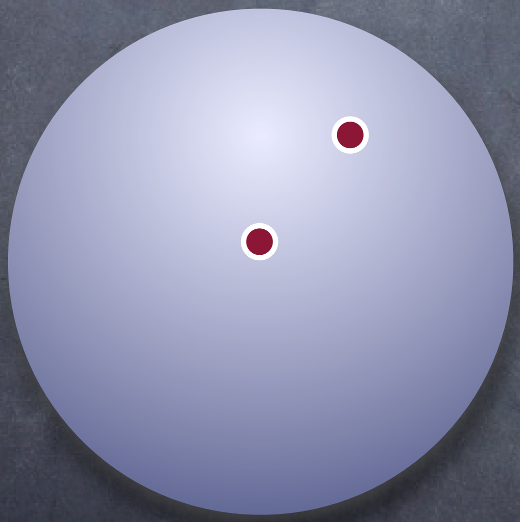

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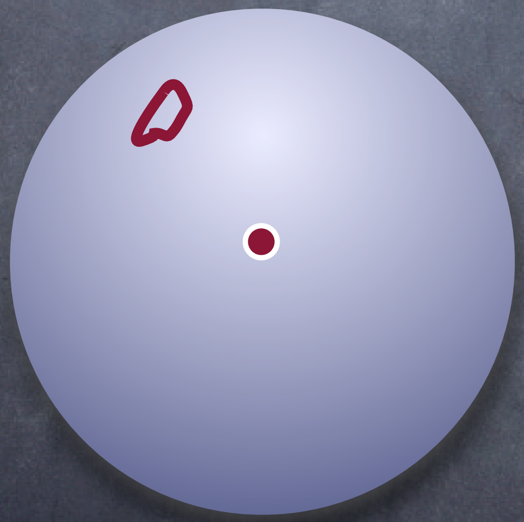

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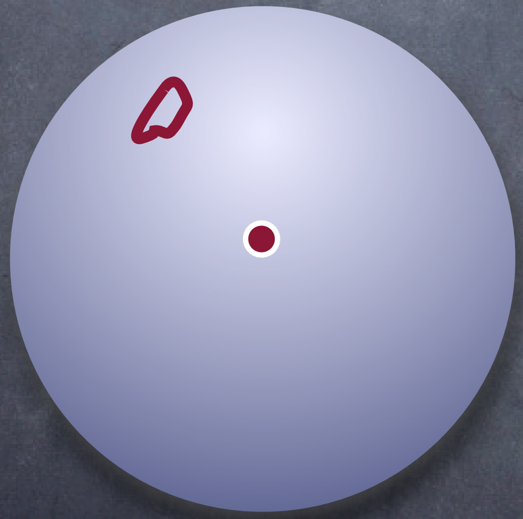

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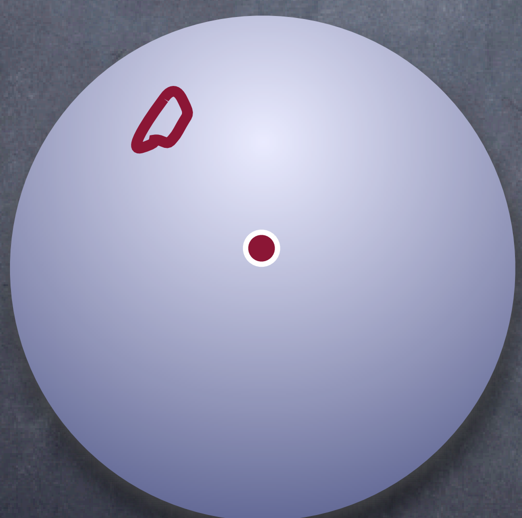

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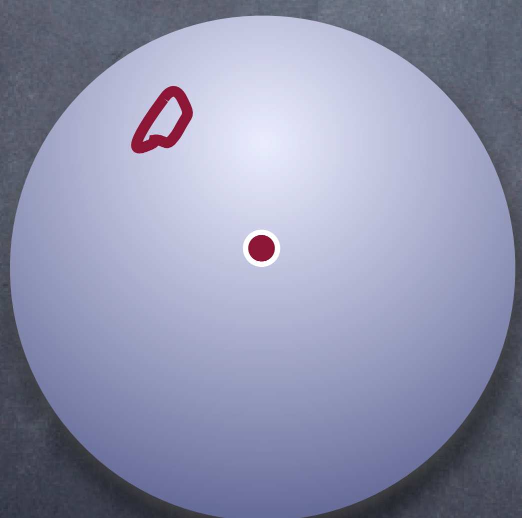

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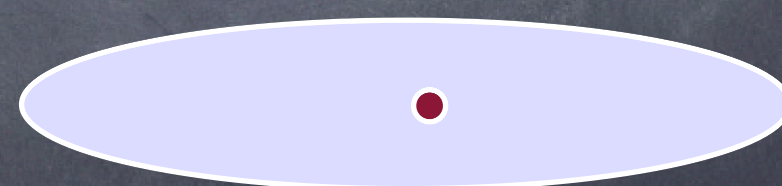

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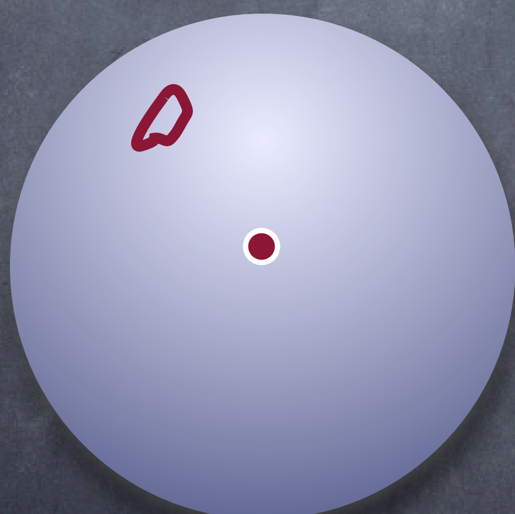

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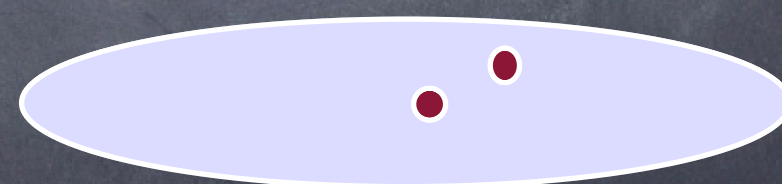

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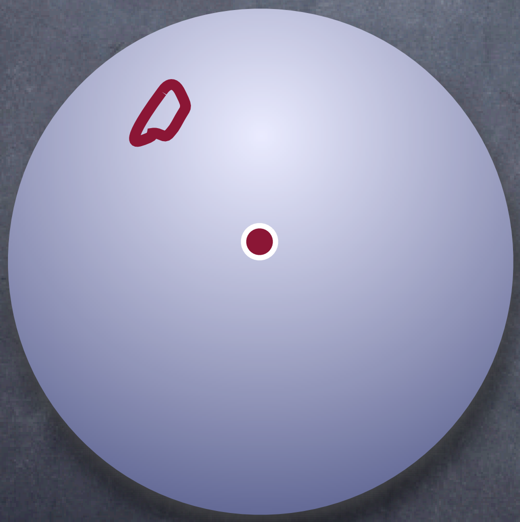

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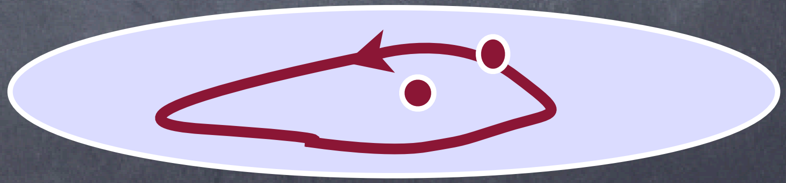

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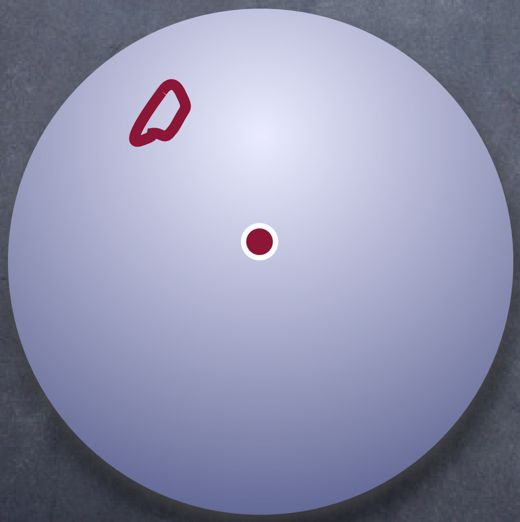

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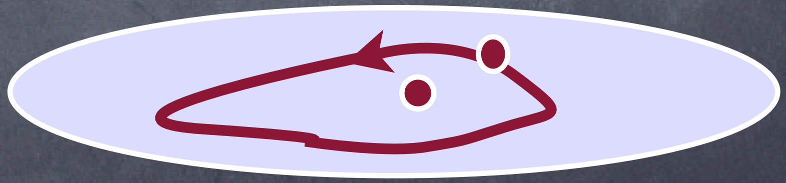

- double exchange $= I$
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- θ can be arbitrary
- (abelian) Anyon

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$d(2n) = 2^{n-1}$ for MR state or p+ip SC

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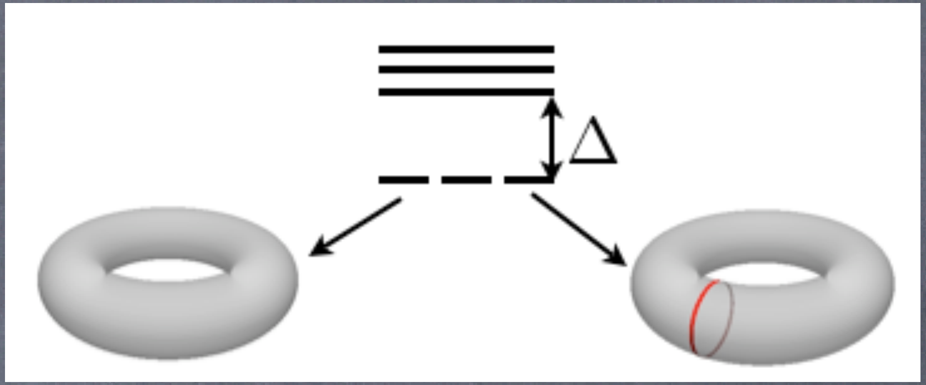
  $N_g=q^g$ e.g., $N_1=3$



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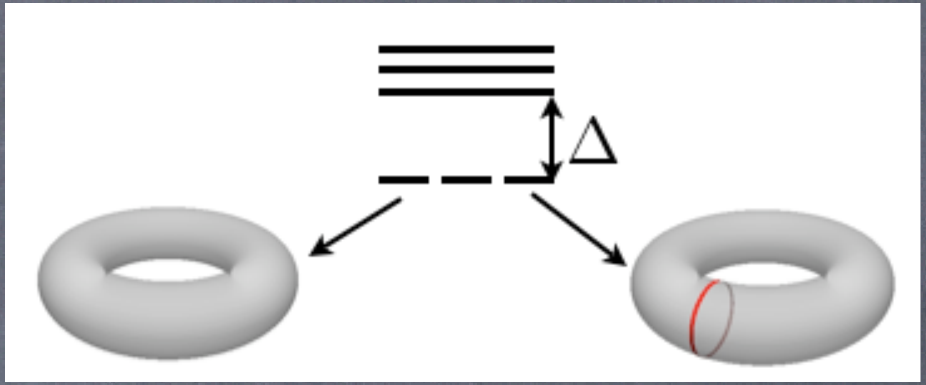

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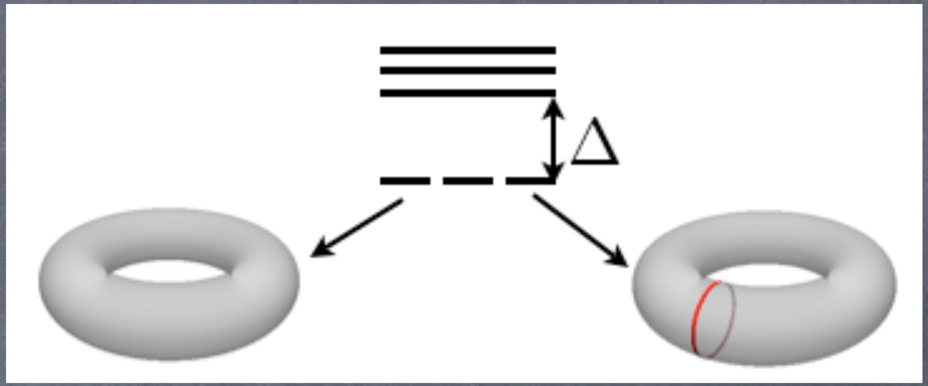

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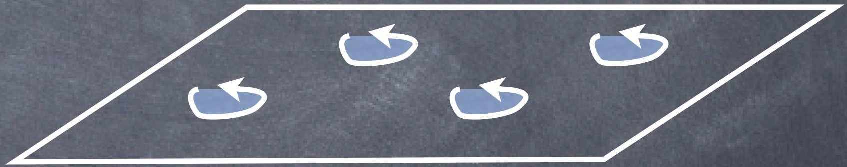

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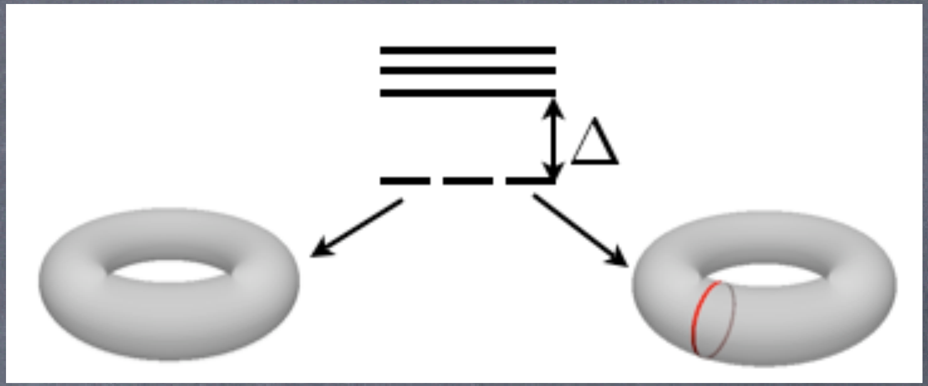

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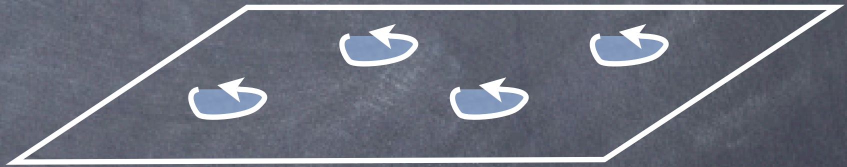

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- Stability of 1/2-qv's in SrRuO
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FQH and Topology

- R_H as a topological invariant:
  - ➡ precise and robust
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  - *e.g., Moore-Read state*
    - $\nu = 5/2$
    - *qp types: 1, σ ($e^*=1/4$), ψ*
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  - ✓ $\nu = 5/2$ point contact operation, e/4 charge

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WANTED: signature of non-abelian anyons

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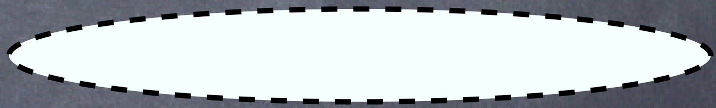

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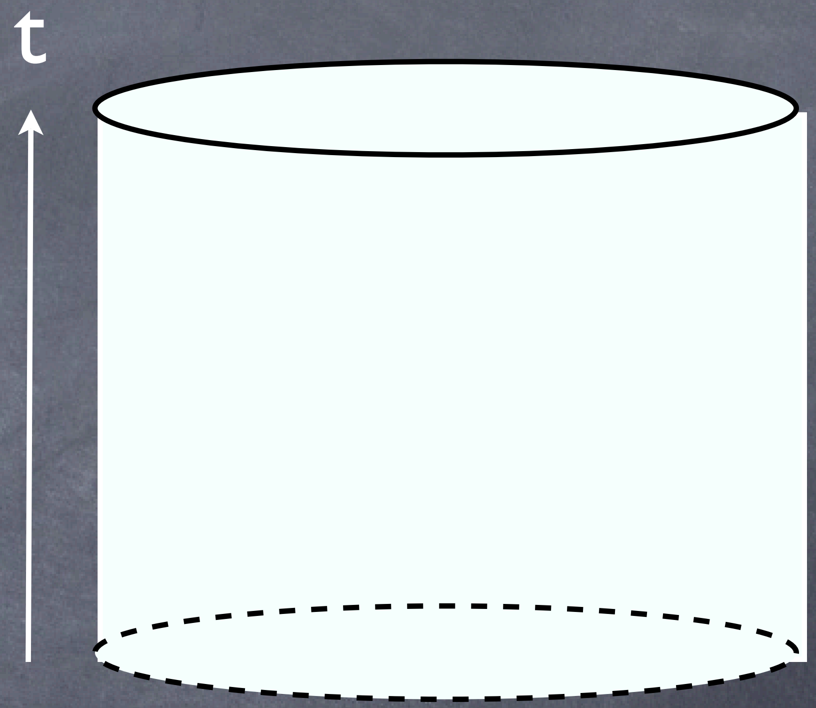

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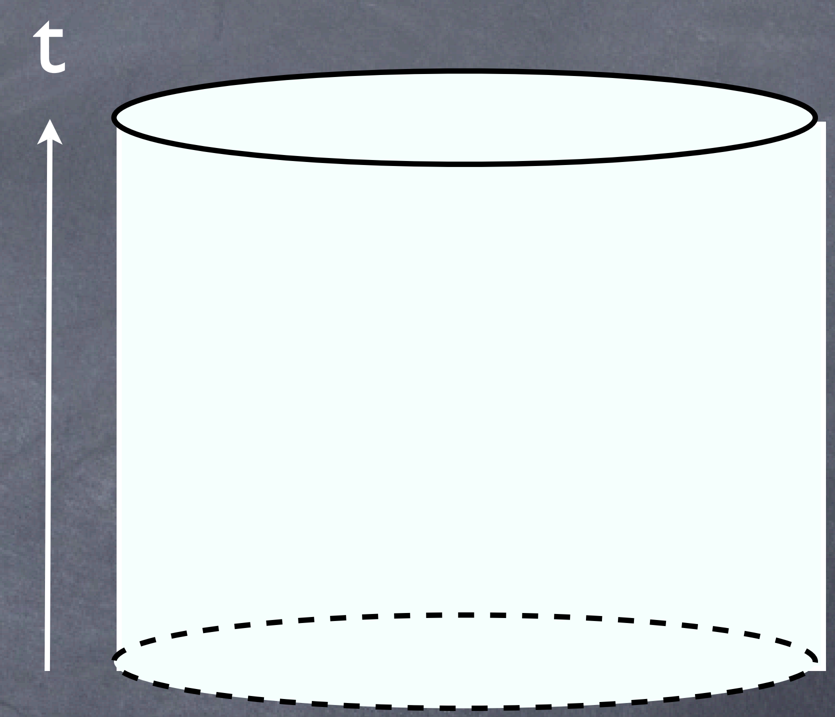

The CS theory and Wilson lines

- the Chern-Simons theory:
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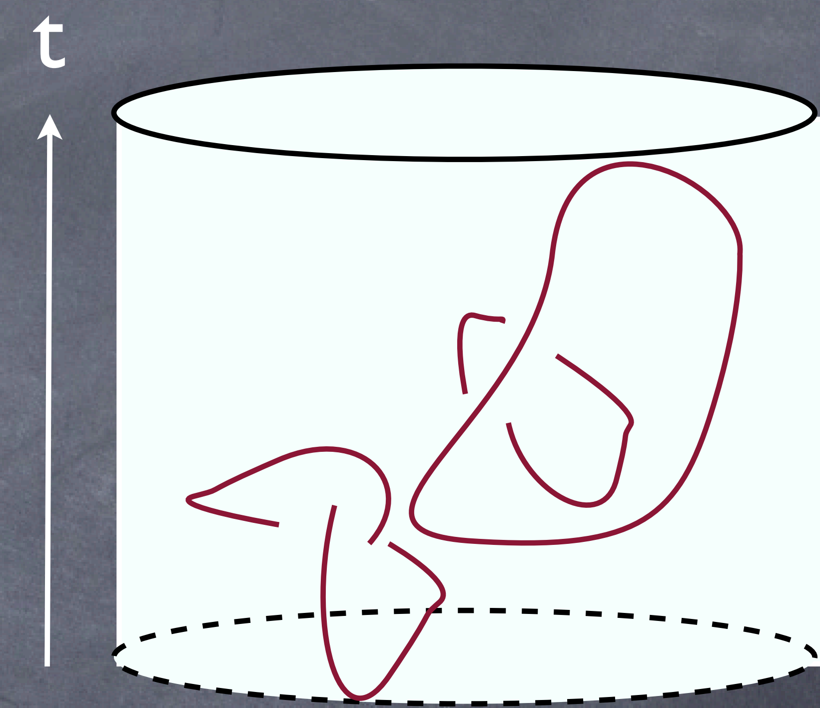

Wilson line configurations in 2+1 D space-time

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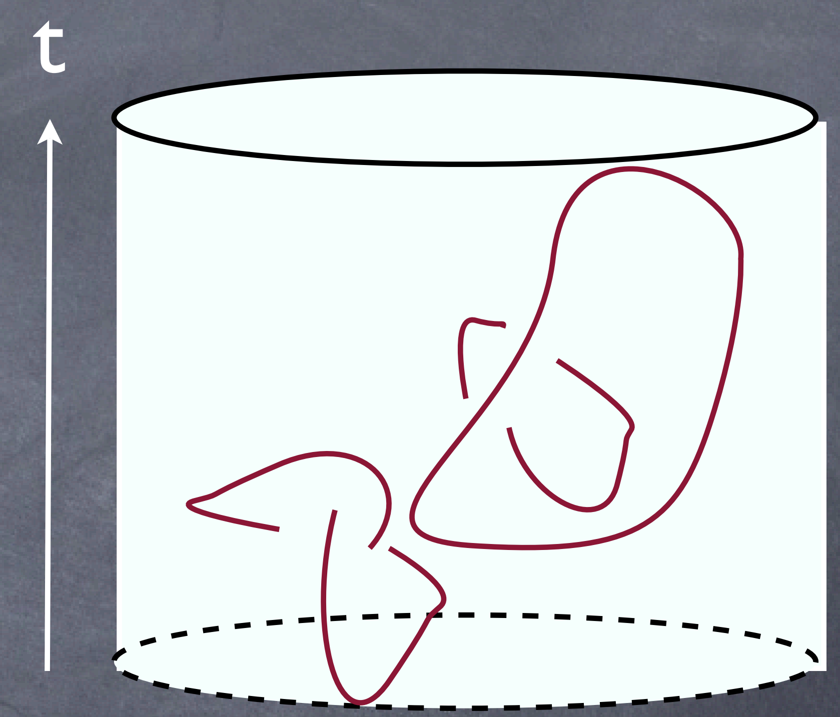

Wilson line configurations in 2+1 D space-time

2+1 D permits nontrivial knots

Where it all started

- Witten: Quantum Field Theory and Jones Polynomial

In a lecture at the Hermann Weyl Symposium last year [1], Michael Atiyah proposed two problems for quantum field theorists. The first problem was to give a physical interpretation to Donaldson theory. The second problem was to find an intrinsically three dimensional definition of the Jones polynomial of knot theory. These two problems might roughly be described as follows.

As for the Jones polynomial and its generalizations [5–11], these deal with the mysteries of knots in three dimensional space (figure 1). The puzzle on the mathematical side was that these objects are invariants of a three dimensional situation, but one did not have an intrinsically three dimensional definition. There

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:knots & representation of compact gauge group

–Cutting three manifold $\mathcal{M}$ with a Riemann surface Σ

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- Fractional quantum Hall effect:

–CS theory is the effective field theory

Jones Polynomial & Wilson lines

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$$\int D\alpha \mathrm{T}r_{1/2}\left\{\mathcal{P}\exp\left[i\oint_\gamma \alpha\right]\right\}\exp\left[\frac{2}{4\pi}\int d^3x\epsilon\mu\nu\lambda\left(\alpha^a_\mu\partial_\nu\alpha^a_\lambda+\frac{2}{3}f_{abc}\alpha^a_\mu\alpha^b_\nu\alpha^c_\lambda\right)\right]$$
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- $V_\gamma(q)$ topological invariant of a knot
- Quantum mechanical amplitudes' dependence on the braiding of world lines

Edge states as boundary theory

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- Finite size sample ⇔

  cutting 2+1D space with a 1+1D surface

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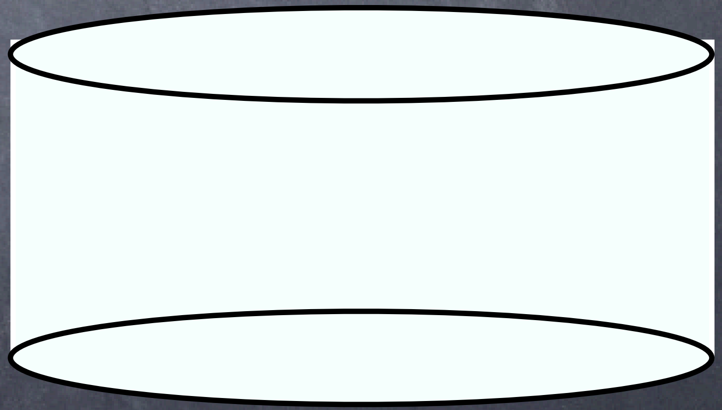

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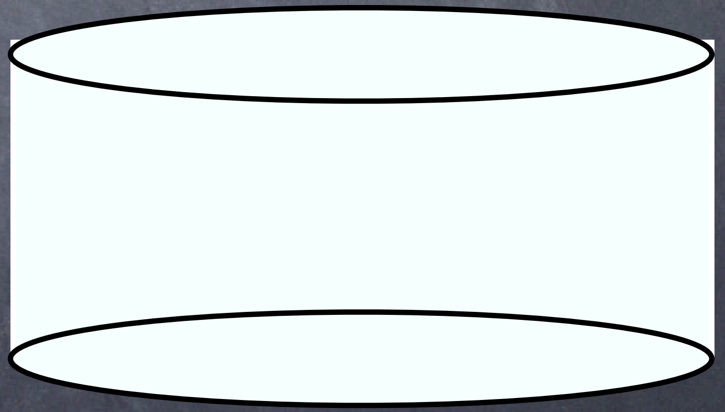

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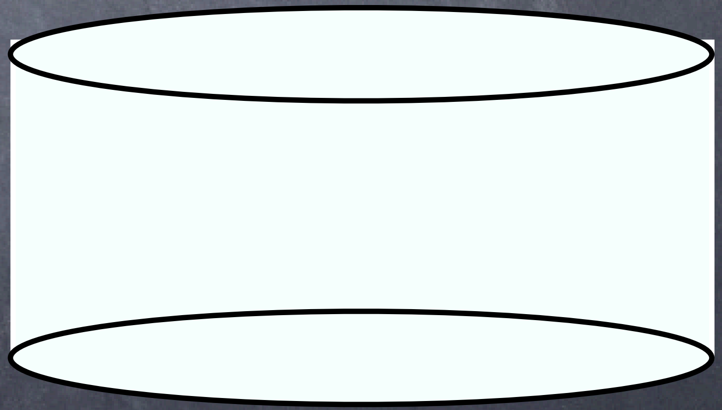

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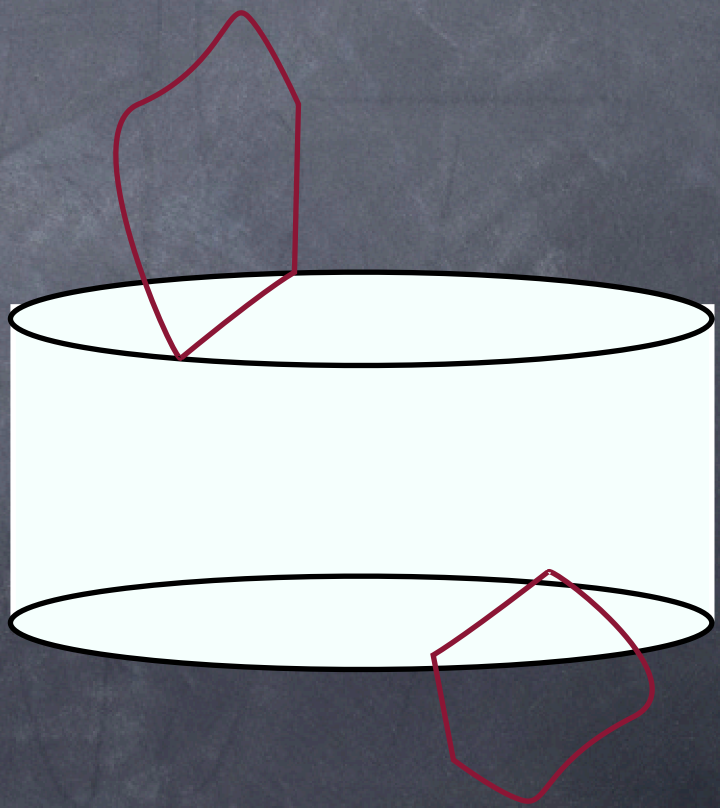

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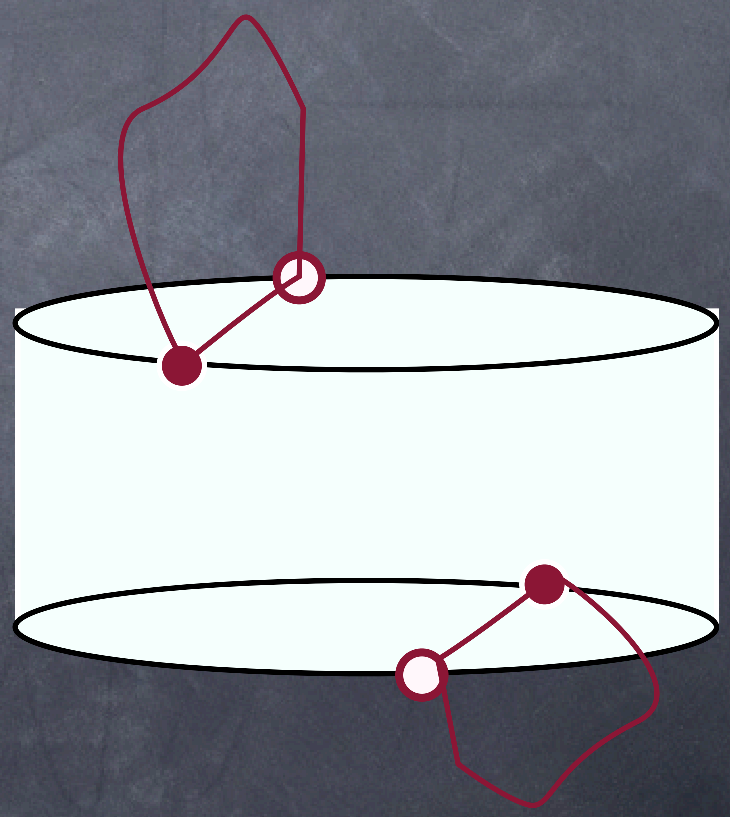

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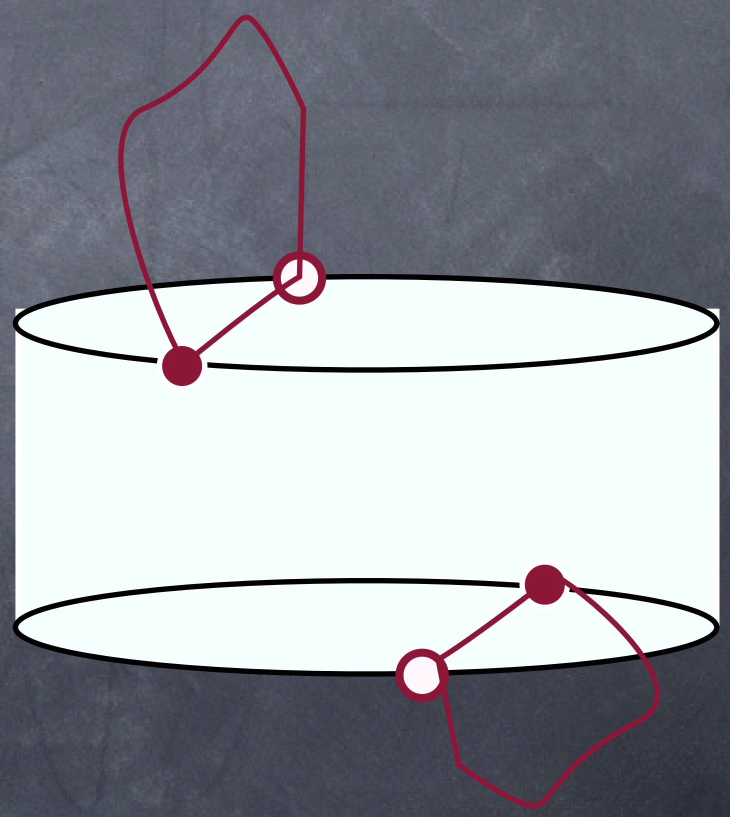

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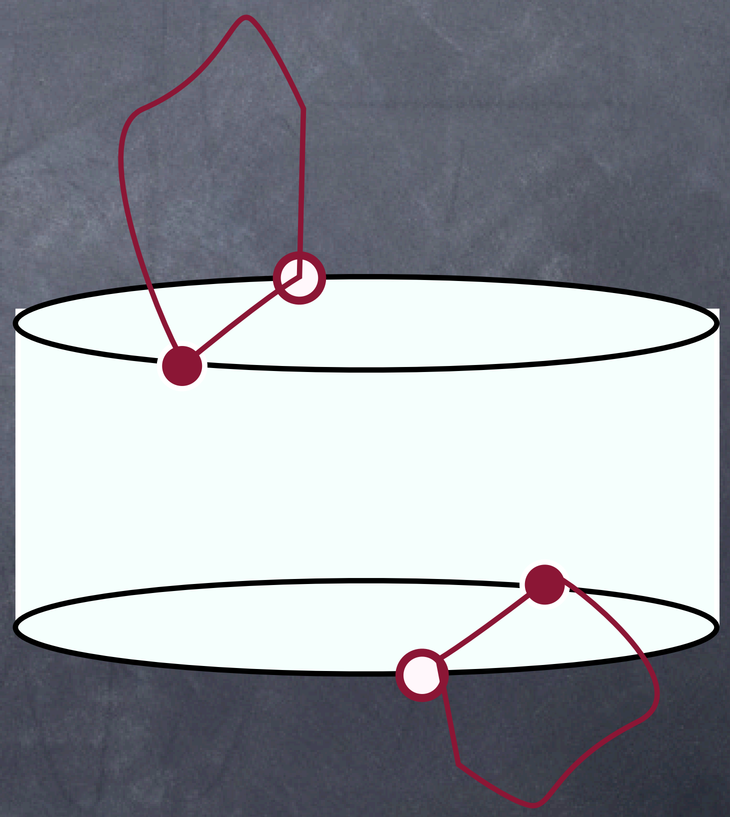

Hydrodynamic picture

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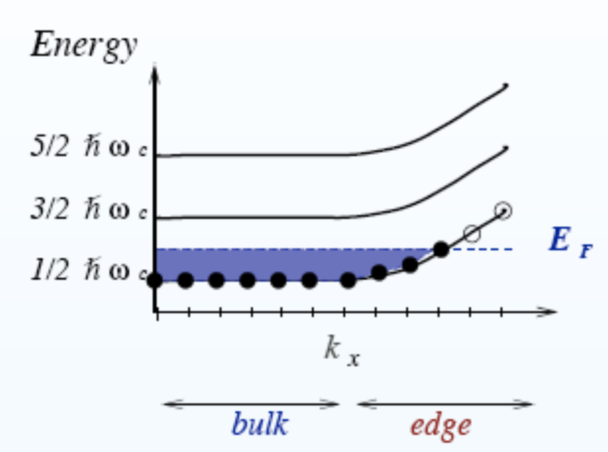

Hydrodynamic picture

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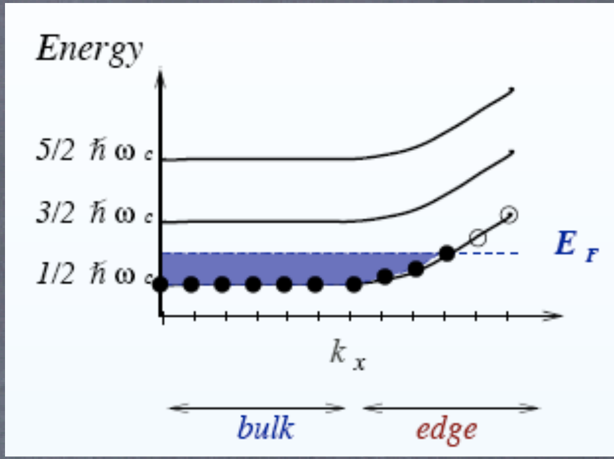

- Dissipationless propagation of 1D density ripples

- Links topology and measurements.

Hydrodynamic picture

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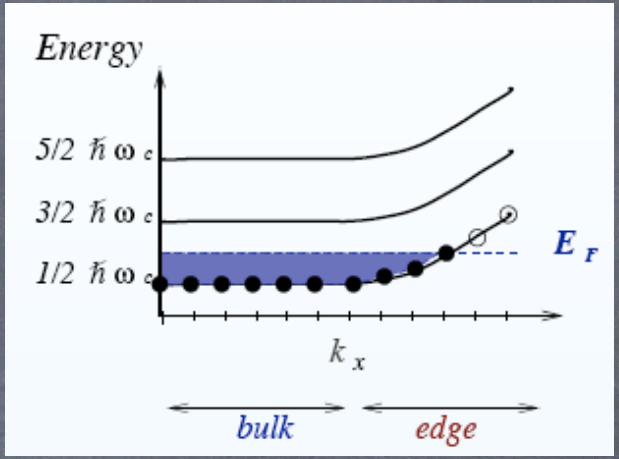

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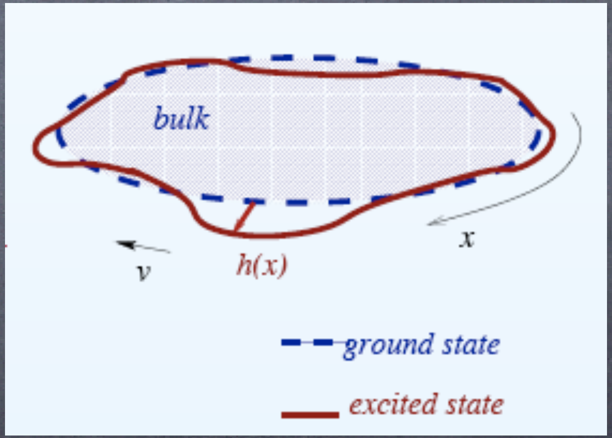
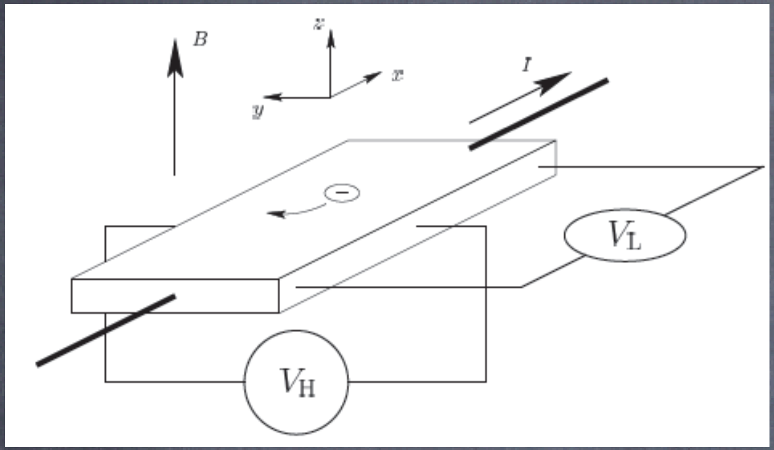


- Links topology and measurements.

Real life?

Transport measurements of QHE

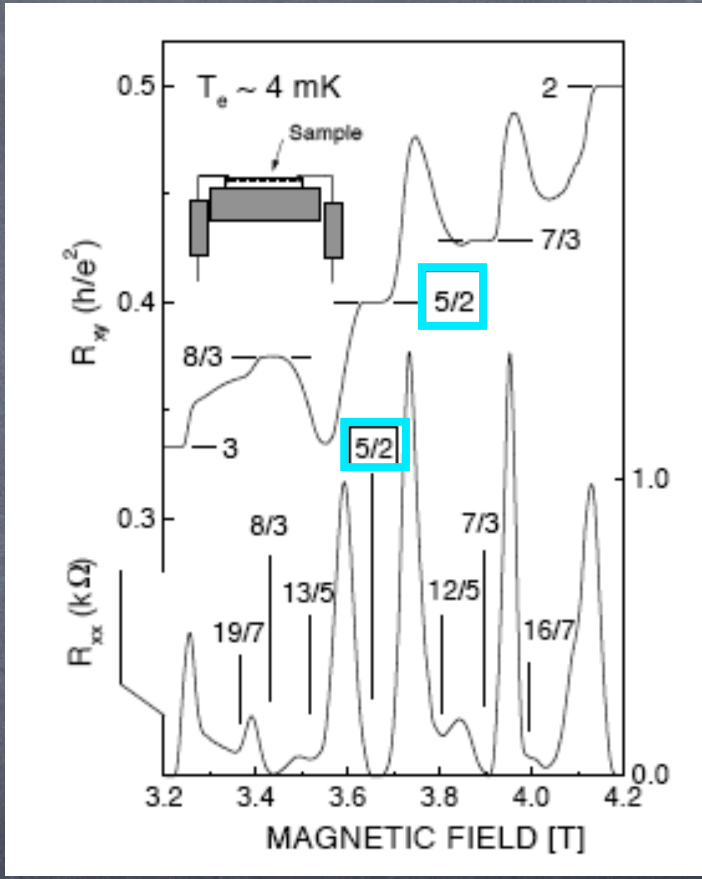
- Quantum Hall setting



-2D electron system

-low T, large magnetic field ~ 10 T

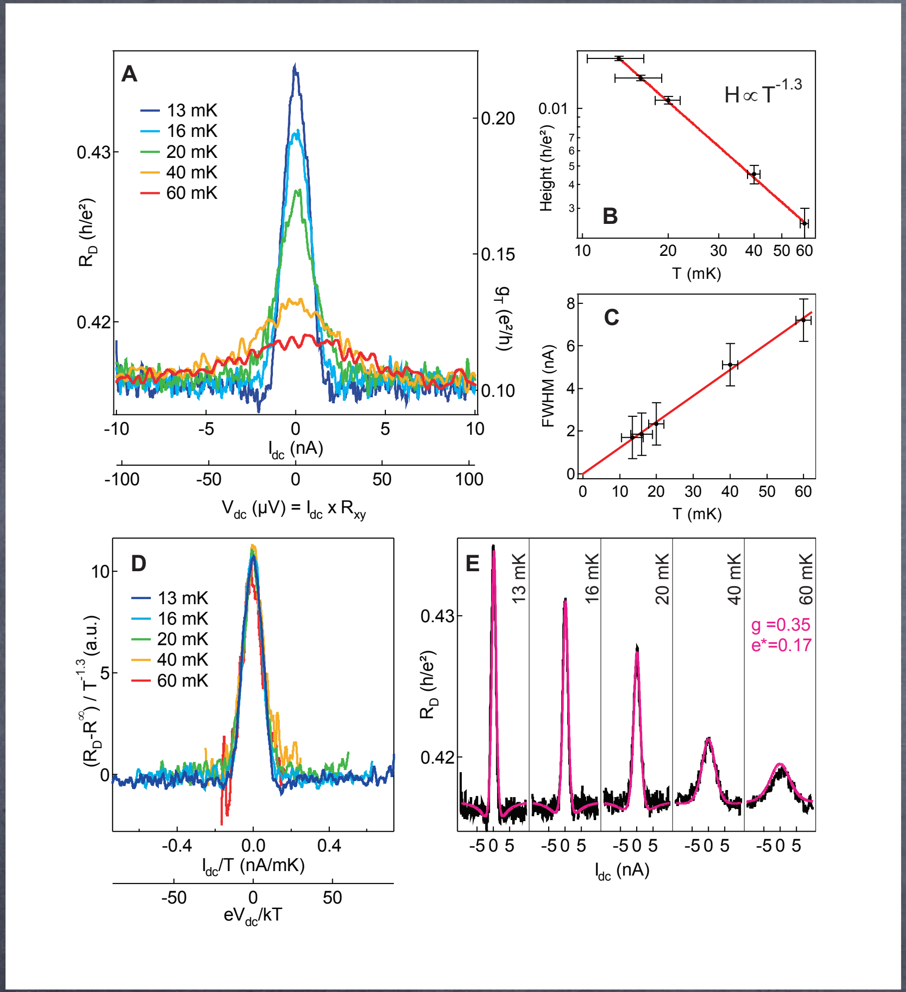
- Quantized magnetotransport

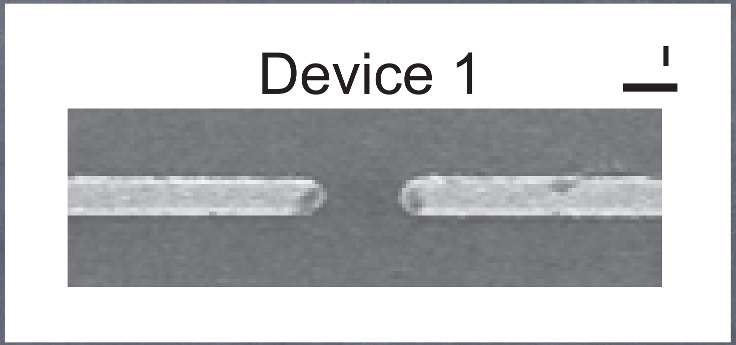


(from W. Pan et al, 1999)

-Robust and precise quantization $R_H = h/\nu e^2$

PC tunneling: poking at the edge states





Radu et al, ArXiv: 0803.3530

Chiral Luttinger liquid behavior

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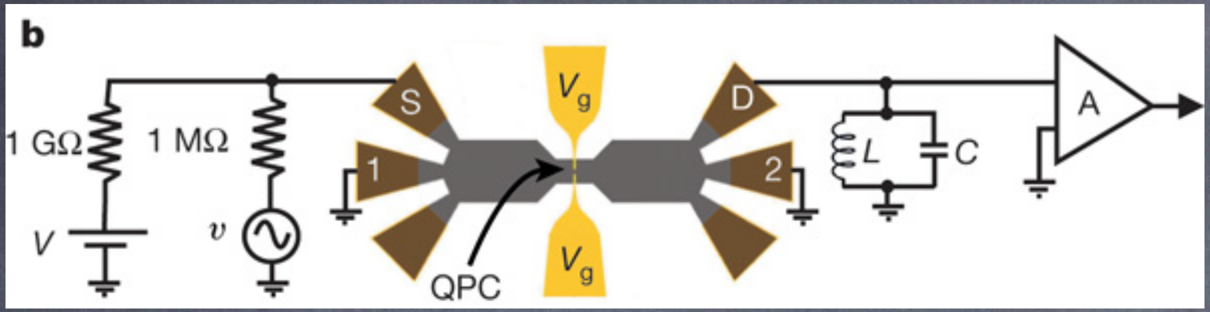
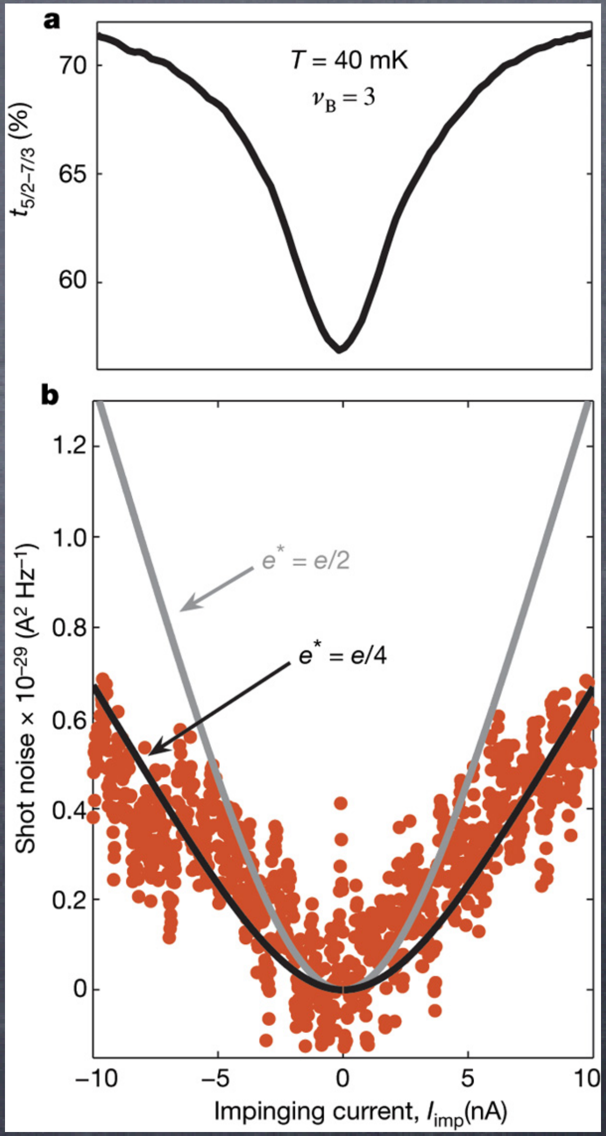


Dolev et al, Nature vol 452, 829 (2008)

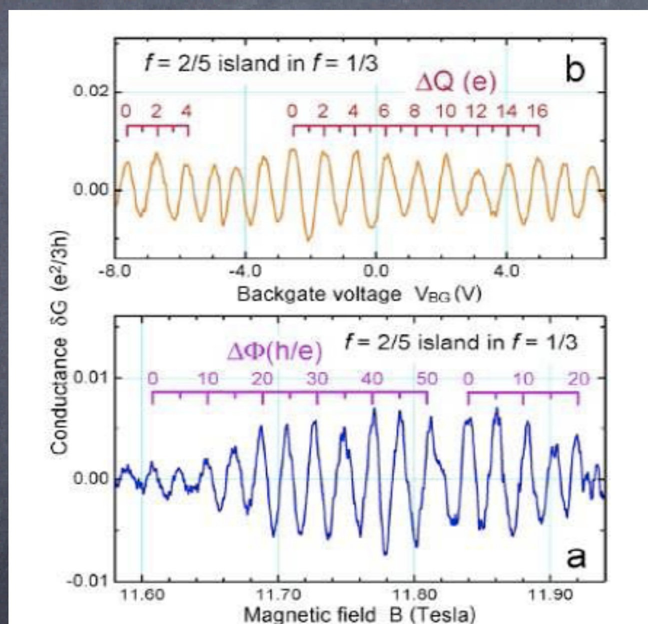
Fractional charge



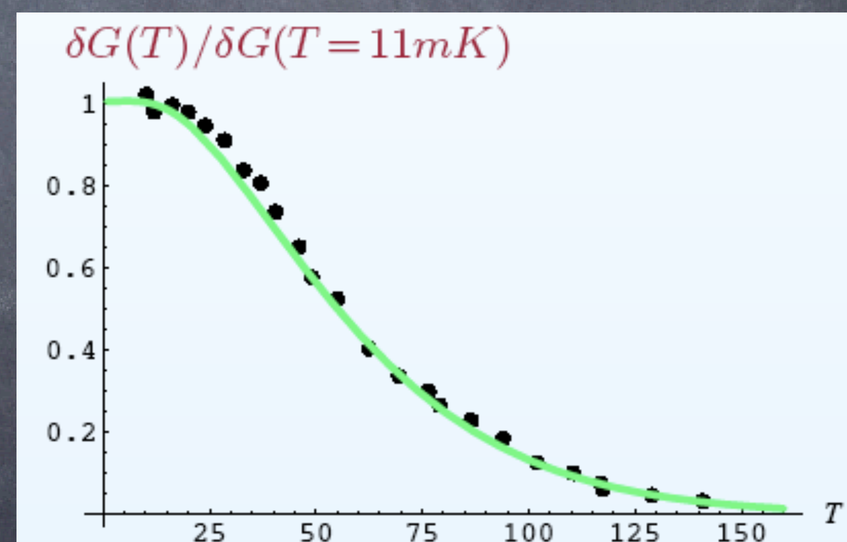
Double PC interferometer

- Success in the abelian case $\nu = 1/3$

- V. Goldman (2005)



- Theory (E.-A. Kim, 2006)
  - correct superperiod
  - T-dep oscillation amplitude



Proposal for MR state

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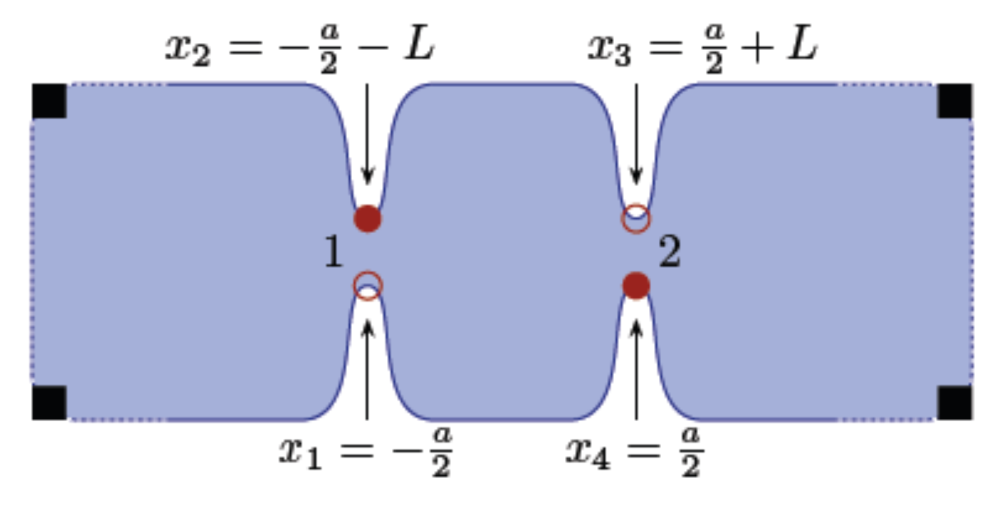
- The interferometer setup



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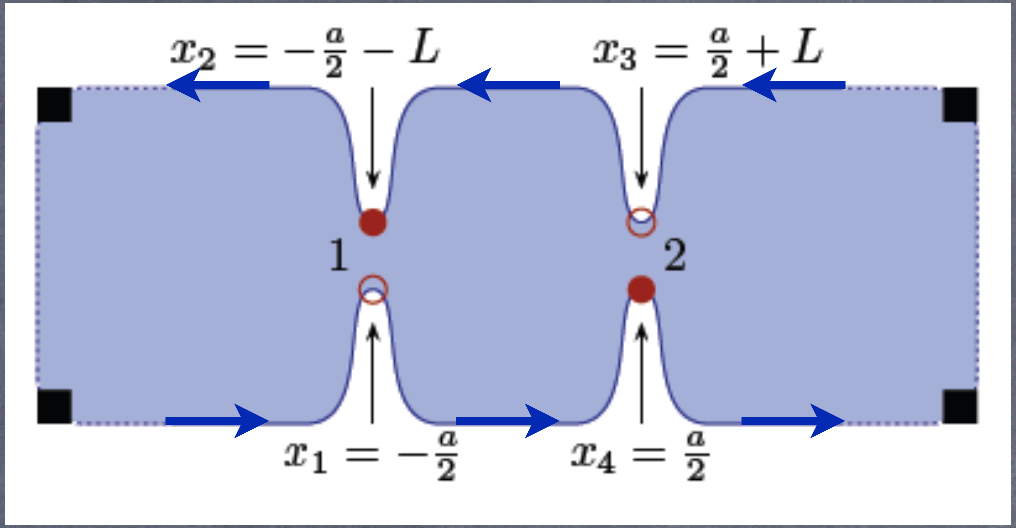

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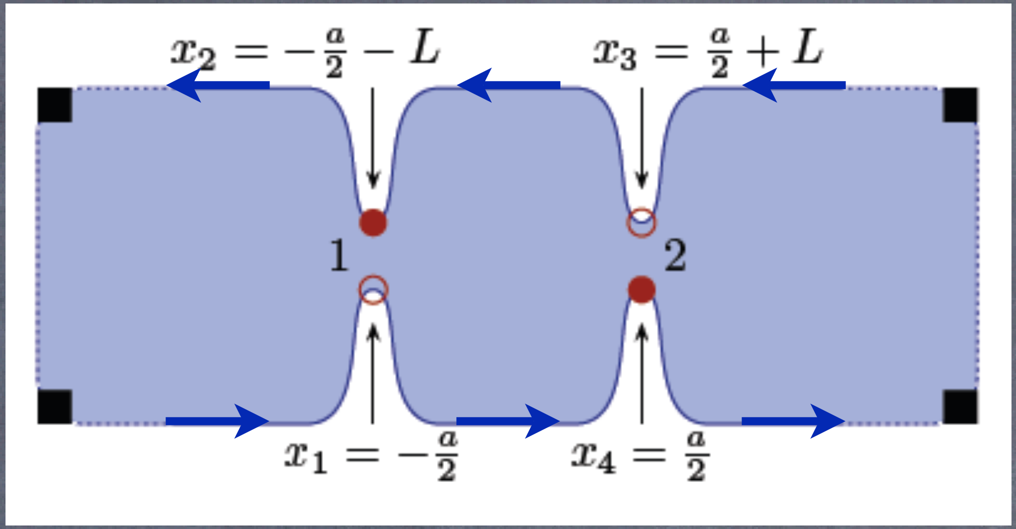

- Perturbative calculation of tunneling response: current $\langle I \rangle$ and noise $\langle S(\omega) \rangle$

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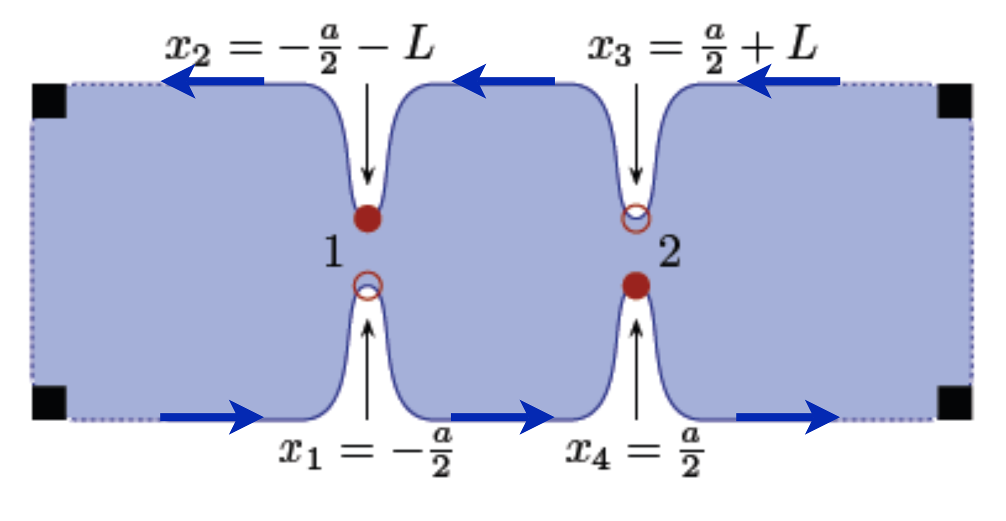

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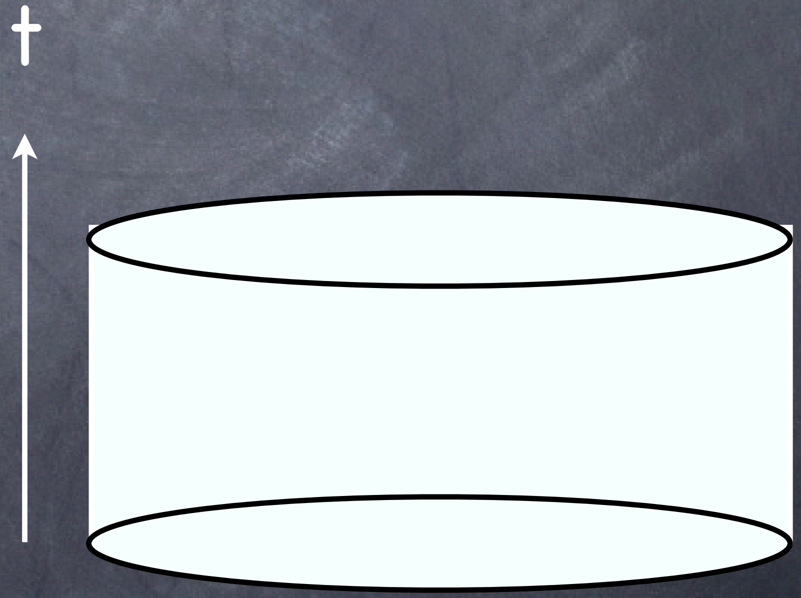
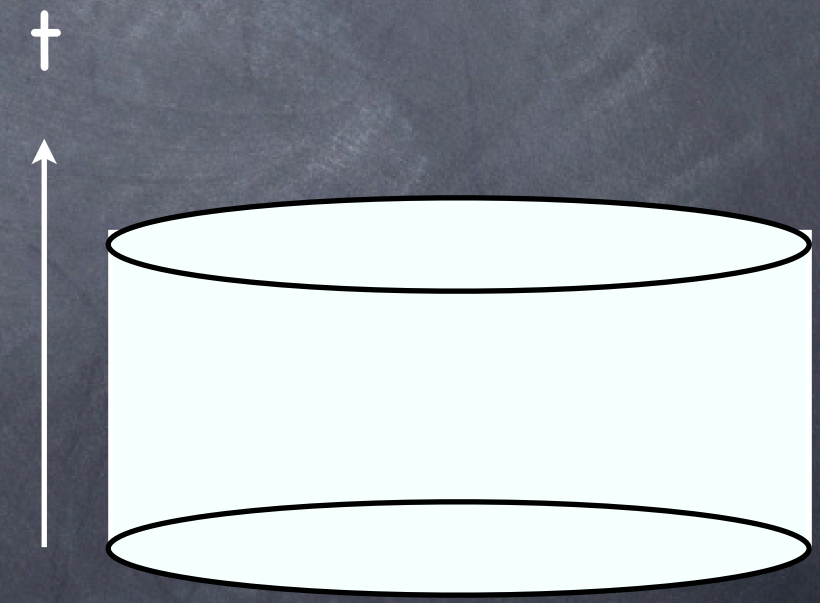
➡ involves a pair of Wilson lines terminating at four marked points

What is tunneling ?



What is tunneling ?

- An instantaneous creation of ptl-hole pair
- Marks two equal time points in 1+1D event space



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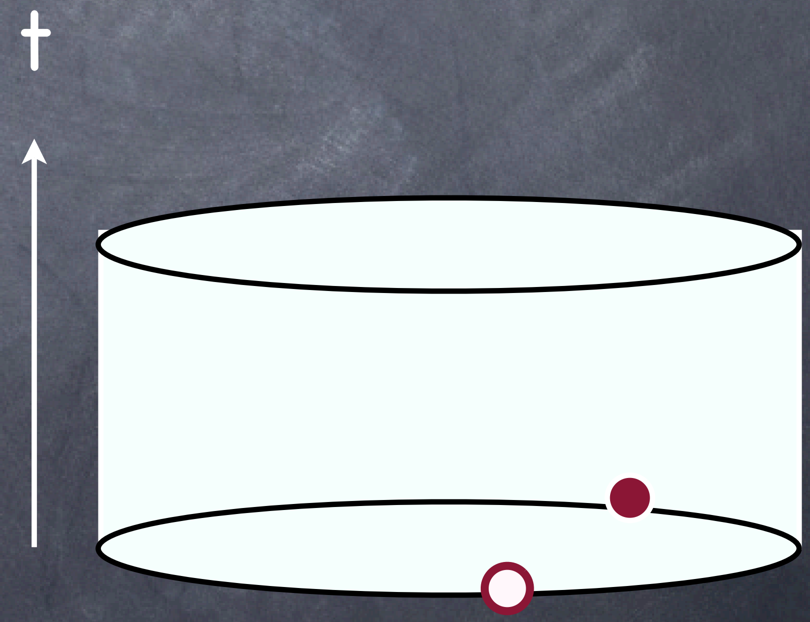

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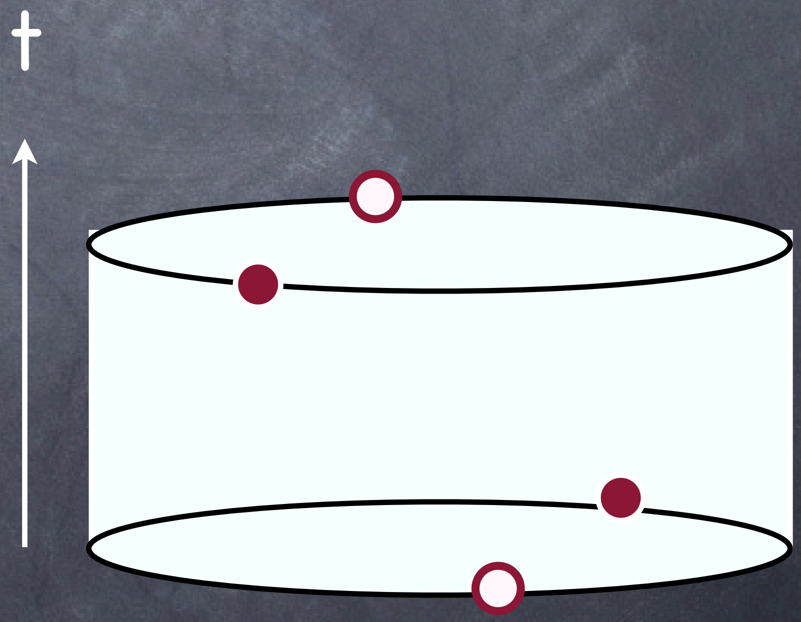

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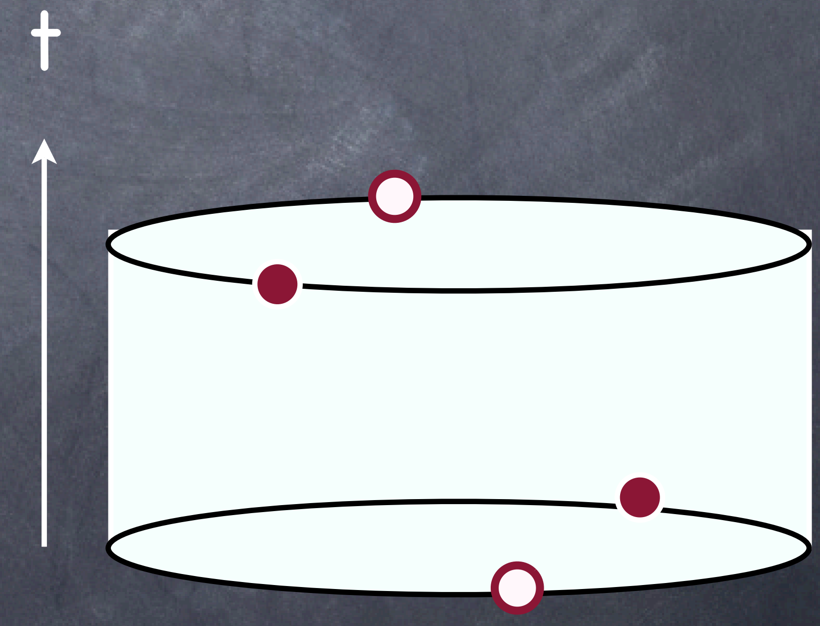

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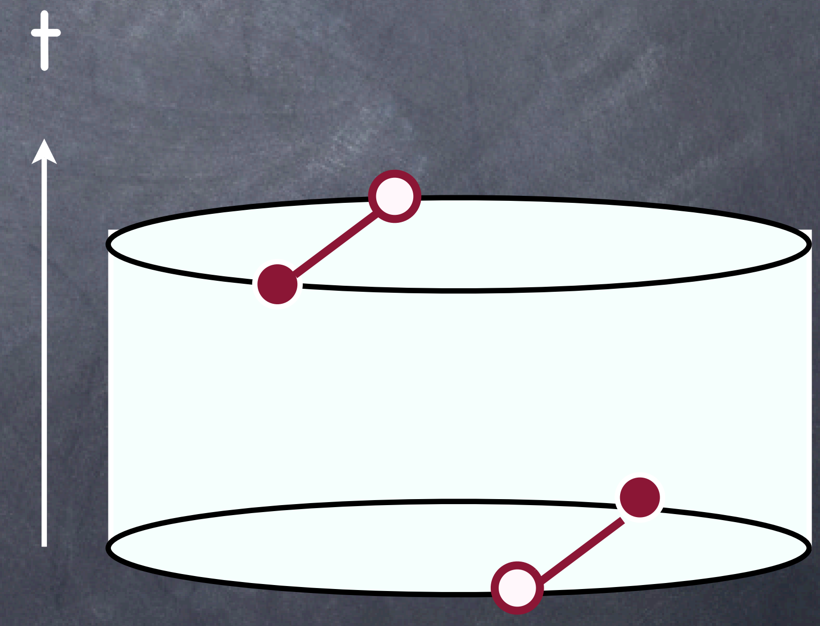

The topology of Wilson line config's.

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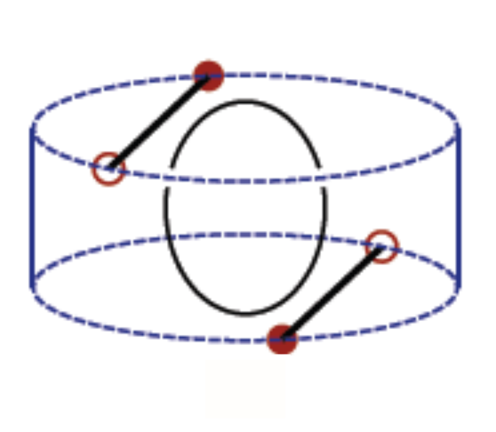

(0)

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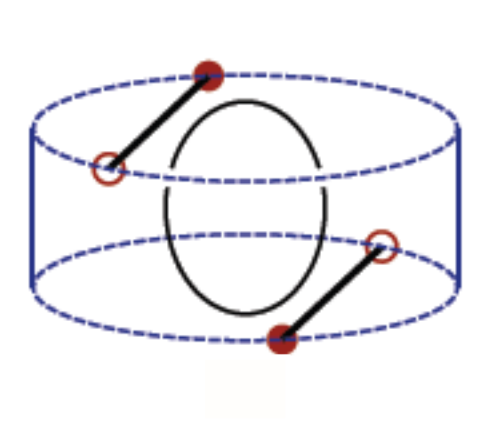

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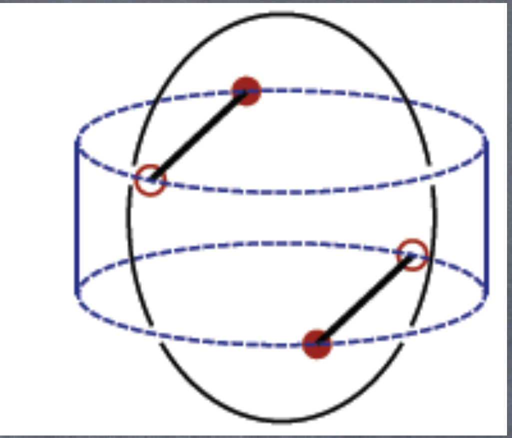

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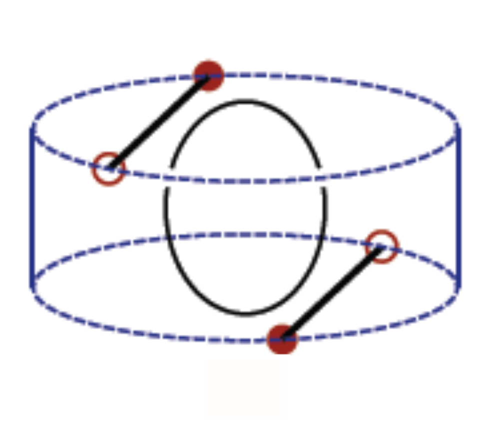

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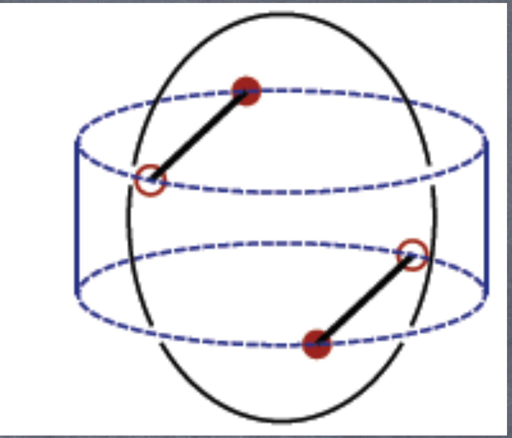

(1)

- Different FQH states ⇔ Different rules for untangling or pulling a Wilson line through another.

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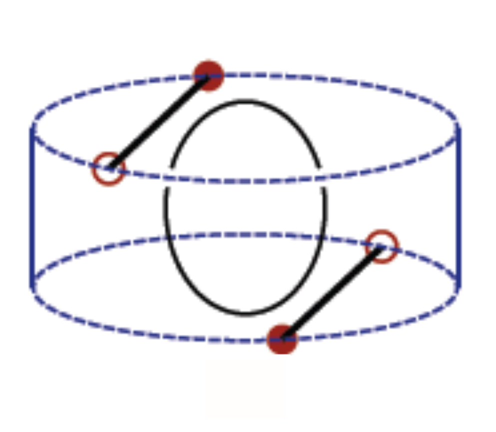

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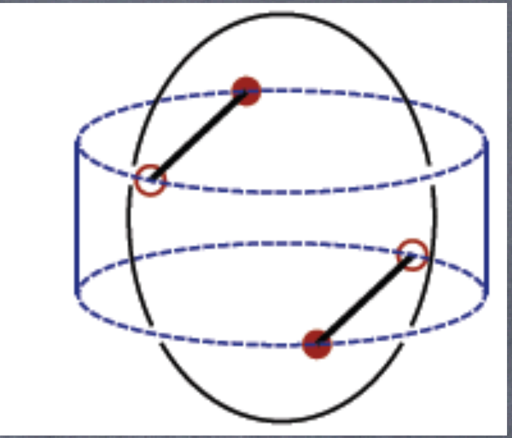


(1)

- Different FQH states ⇔ Different rules for untangling or pulling a Wilson line through another.
- Lines can, effectively, be pulled through for abelian states

Can these two possibilities show up in measurable quantities for the MR state ?

The current v.s. the noise

- Current defined as a response

$$\langle \hat{I}(t) \rangle = -i \int_{-\infty}^{t} dt' \langle [\hat{I}(t), \hat{H}_{\rm tun}(t')] \rangle$$

commutator

- Noise (fluctuation)

$$S(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} dt' e^{i\omega t'} \langle \{ \hat{I}(t), \hat{I}(t') \} \rangle$$

anti-commutator

Edge state theory

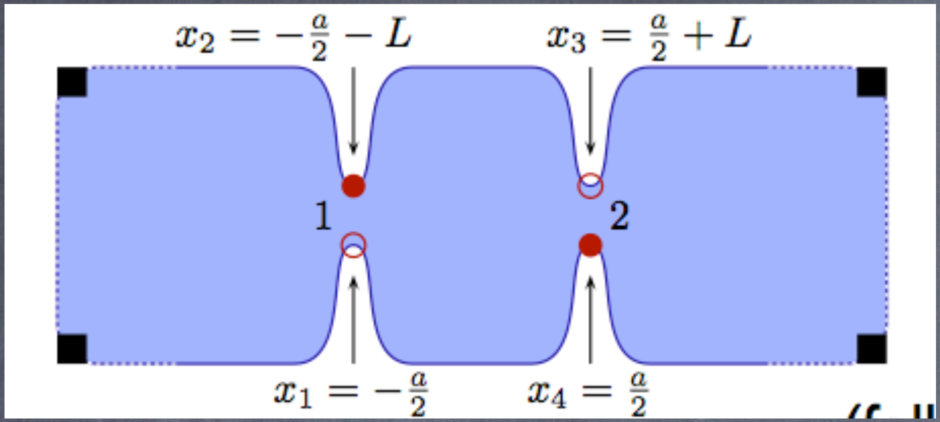
- The charge: chiral boson ϕ_c

$$\mathcal{L}_c = 1/(2\pi)\partial_x\varphi_c(\partial_t + v\partial_x)\varphi_c$$

- Non-Abelian statistics: Ising conformal field theory
  - Primary fields 1, ψ, σ
  - Electron operator: $V_{\mathrm{el}}(z) \quad = \quad \psi(z)e^{i\sqrt{2}\varphi_c}(z)$
  - Quasi-particle operator: $V_{\mathrm{qh}}(z) \quad = \quad \sigma(z)e^{i/\sqrt{8}\varphi_c}(z)$

,where $z = i(vt - x)$

double PC calculation



(after C. Chamon et al.)

- PC tunneling: instantaneous charge transfer

$$\begin{aligned}\hat{V}_1(t) &= \sigma(x_1,t)\sigma(x_2,t)e^{i/\sqrt{8}\varphi_c(x_1,t)}e^{-i/\sqrt{8}\varphi_c(x_2,t)}\\ \hat{V}_2(t) &= \sigma(x_3,t)\sigma(x_4,t)e^{i/\sqrt{8}\varphi_c(x_3,t)}e^{-i/\sqrt{8}\varphi_c(x_4,t)}\end{aligned}$$

- Tunneling Hamiltonian

$$\hat{H}_{\text{tun}}(t) = \sum_j \Gamma_j(t)\hat{V}_j(t) + \text{h.c.} \qquad \Gamma_j(t) = \Gamma_j e^{i\omega_0 t} \qquad \omega_0 = \frac{e^* V}{\hbar}$$

current and noise

$$\langle \hat{I} \rangle^{(p)} \equiv \langle \hat{I} \rangle_d^{(p)} + \cos(\tfrac{\phi}{\Phi_0}) \langle \hat{I} \rangle_{\rm osc}^{(p)} = \Re \int_0^\infty dt \sum_{j,k=1}^{2} \Gamma_j \Gamma_k^* \left[e^{-i\omega_0 t} \left(\langle \hat{V}_j \hat{V}_k^\dagger \rangle^{(p)}(t) - \langle \hat{V}_k^\dagger \hat{V}_j \rangle^{(p)}(t) \right) \right]$$

$$\langle S(\omega) \rangle^{(p)} \equiv \langle S(\omega) \rangle_d^{(p)} + \cos(\tfrac{\phi}{\Phi_0}) \langle S(\omega) \rangle_{\rm osc}^{(p)} = \Re \int_{-\infty}^\infty dt \sum_{j,k=1}^{2} \Gamma_j \Gamma_k^* \left[e^{i(\omega-\omega_0)t} \left(\langle \hat{V}_j \hat{V}_k^\dagger \rangle^{(p)}(t) + \langle \hat{V}_k^\dagger \hat{V}_j \rangle^{(p)}(t) \right) \right]$$

Both require four-σ correlator

Ising CFT technology

- Fusion rule: a part of definition of CFT

$$
\begin{aligned}
\psi \times \psi &= \mathbf{1} \\
\sigma \times \sigma &= \mathbf{1} + \psi \\
\sigma \times \psi &= \sigma
\end{aligned}
$$

- four σ correlator:

simplest object displaying non-Abelian nature

$$
\begin{aligned}
\langle \sigma(z_1)\sigma(z_2)\sigma(z_3)\sigma(z_4)\rangle^{(p)} &= \frac{1}{\sqrt{2}}(z_1 - z_2)^{-\frac{1}{8}}(z_3 - z_4)^{-\frac{1}{8}}(1-\xi)^{-1/8} \\
&\quad \times \sqrt{1 + (-1)^p \sqrt{1-\xi}}
\end{aligned}
$$

$$
\xi = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}
$$

- Two channels p=0,1 : the key feature of non-Abelian statistics. (qubit)

The current v.s. the noise

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- Current

The current v.s. the noise

- Current
  - A response to external voltage
  - Causality

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$$\langle \hat{I} \rangle^{(p)}_{\mathrm{osc}} = 4e^* \sqrt{\pi T} |\Gamma_1 \Gamma_2| \int_a^\infty dt \frac{\sin(\omega_0 t)}{\sinh(\pi T(t-a))^{1/4} \sinh(\pi T(t+a))^{1/4}}$$

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$$\begin{aligned} \langle S(\omega) \rangle^{(p)}_{\rm osc} &= 4(e^*)^2 \sqrt{\pi T} |\Gamma_1 \Gamma_2| \times \\ &\Big(\int_a^\infty dt \frac{\cos((\omega+\omega_0)t) + \cos((\omega-\omega_0)t)}{\sinh(\pi T(t-a))^{1/4} \sinh(\pi T(t+a))^{1/4}} + \\ &(-1)^p \int_0^a dt \frac{\sqrt{2}(\cos((\omega+\omega_0)t) + \cos((\omega-\omega_0)t))}{\sinh(\pi T(t-a))^{1/4} \sinh(\pi T(t+a))^{1/4}} \Big) \end{aligned}$$

The current v.s. the noise

- Current
  - A response to external voltage
  - Causality
- Noise
  - Fluctuation (correlation) in current
  - Not bound by causality

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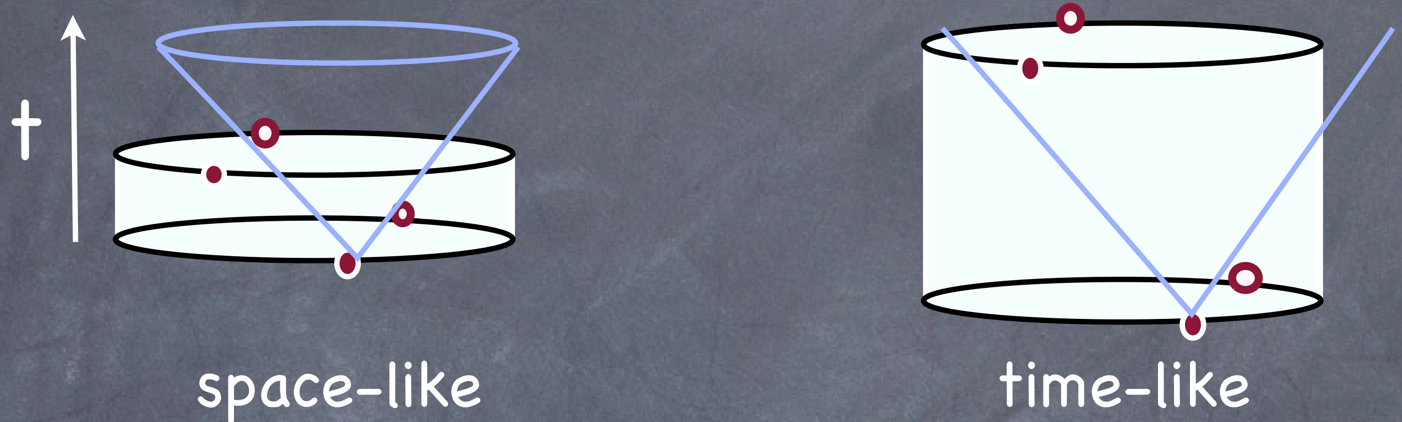
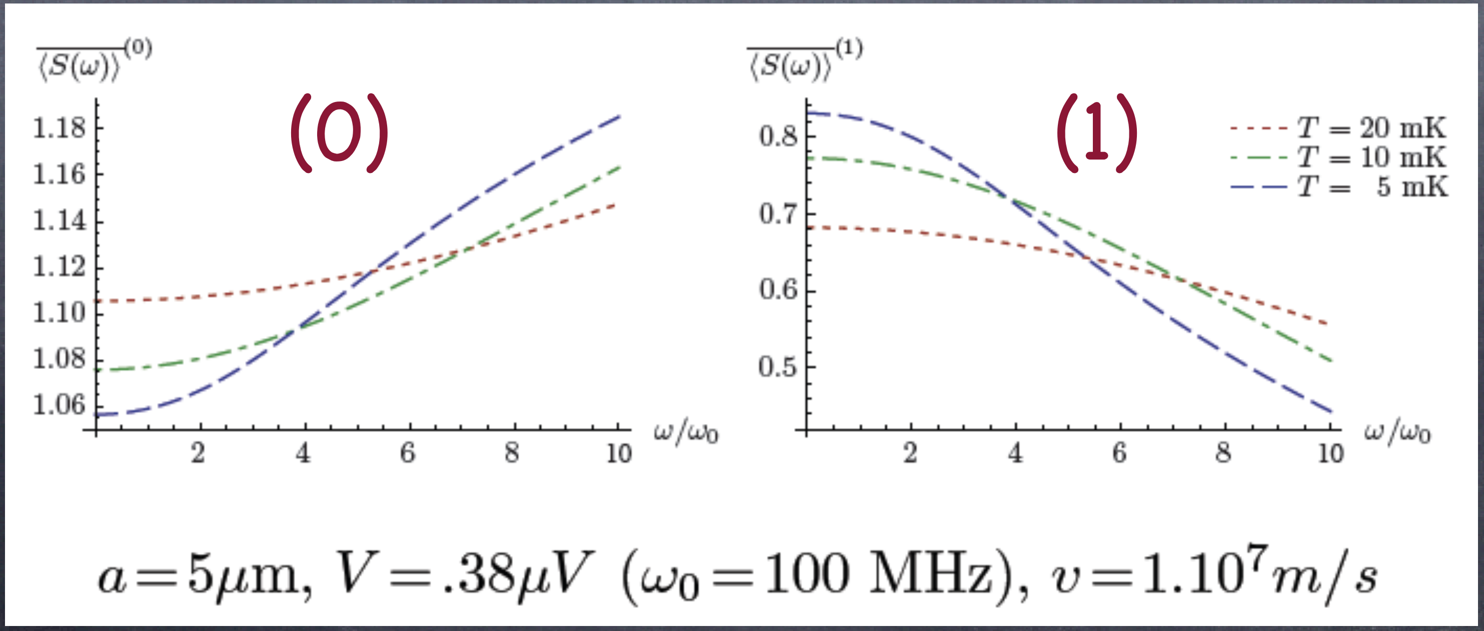
Two states in interference noise

- State dependence only for space-like separation.

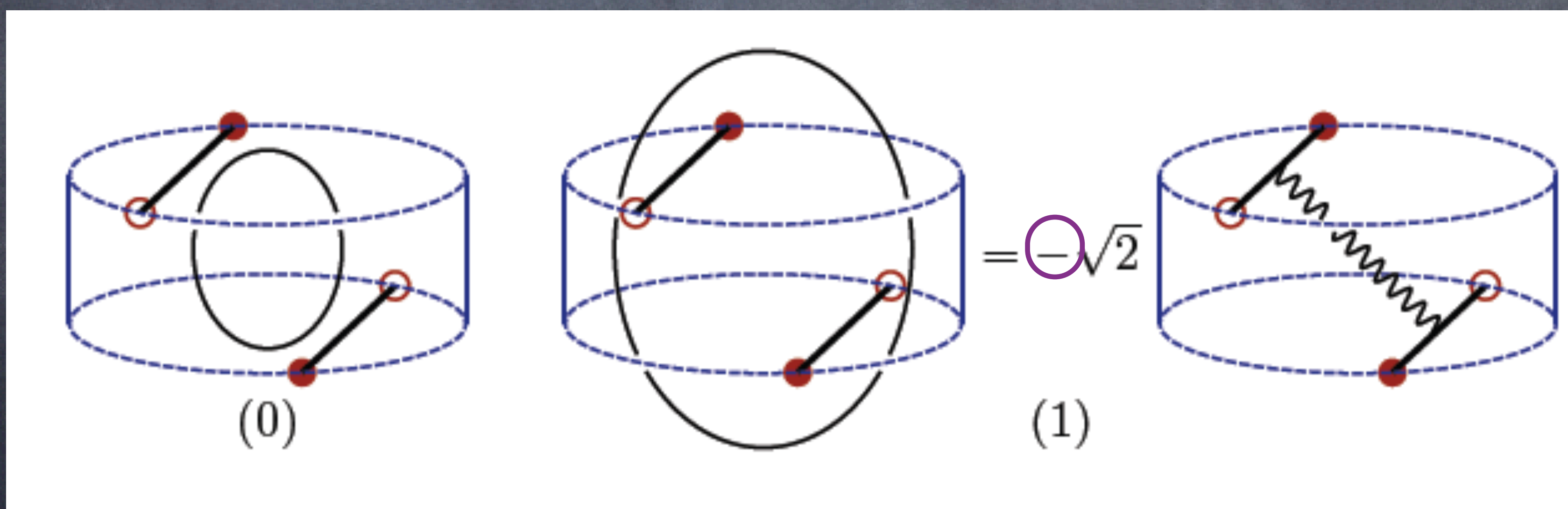


- Interference noise: qualitative state dependence.



Interpretation

- Why only in the non-causal region?
  - ➡ Non-local entanglement can only be seen by space-like separated events



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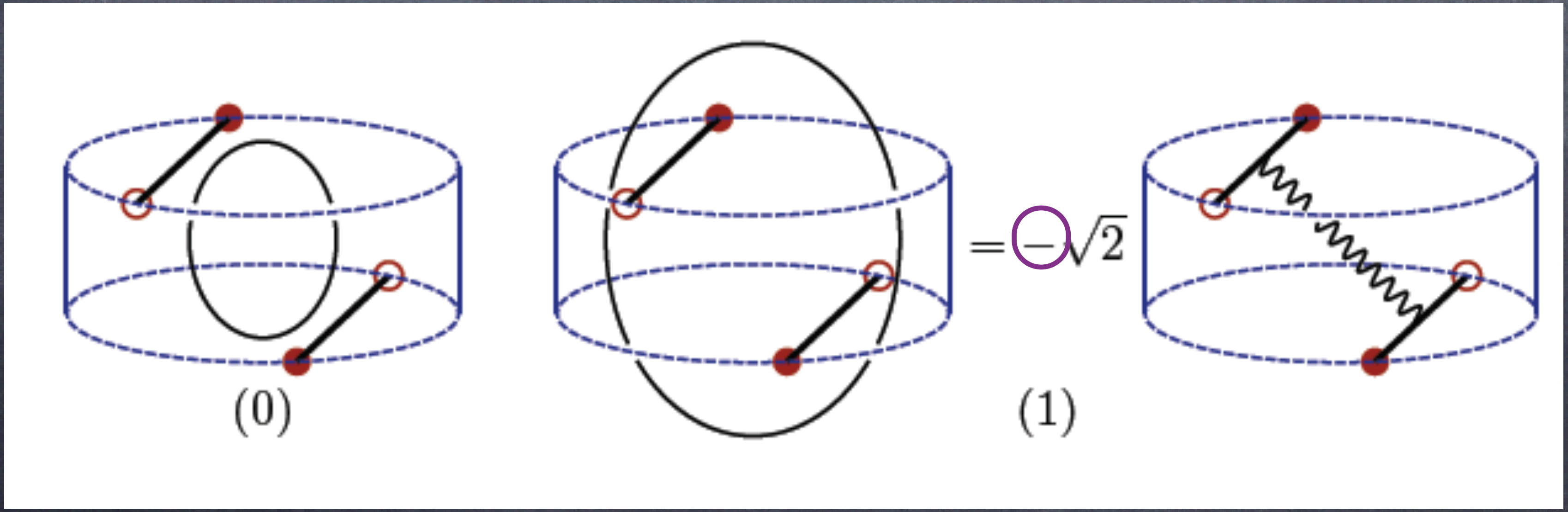
- Why decreasing function of ω in the state (1)



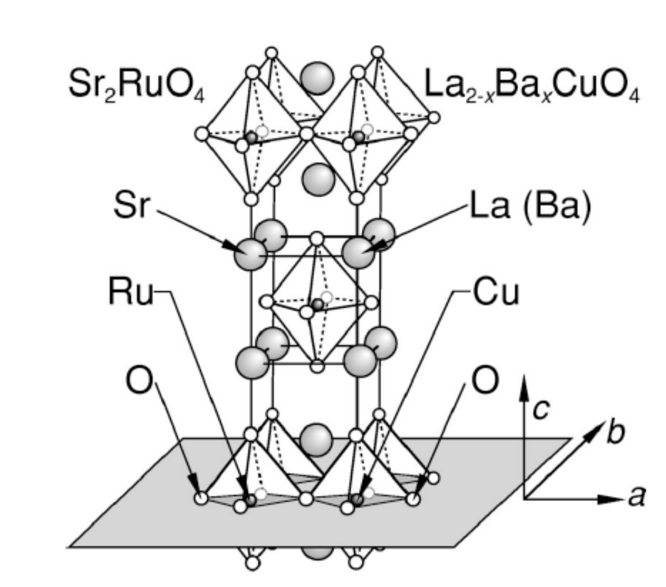
In search of topological states with fractionalized excitations.

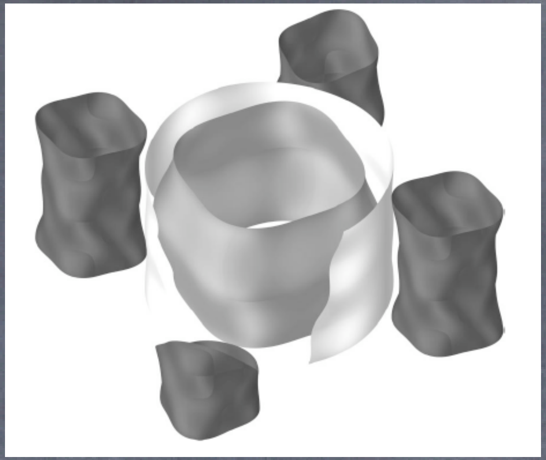
- Topological order and fractionalization
- Poking at Fractional quantum Hall states
- Stability of 1/2-qv's in SrRuO
- Summary and outlook

K. Ishida et al, Nature (1998)

Spin-triplet superconductivity in Sr2RuO4 identified by 17O Knight shift

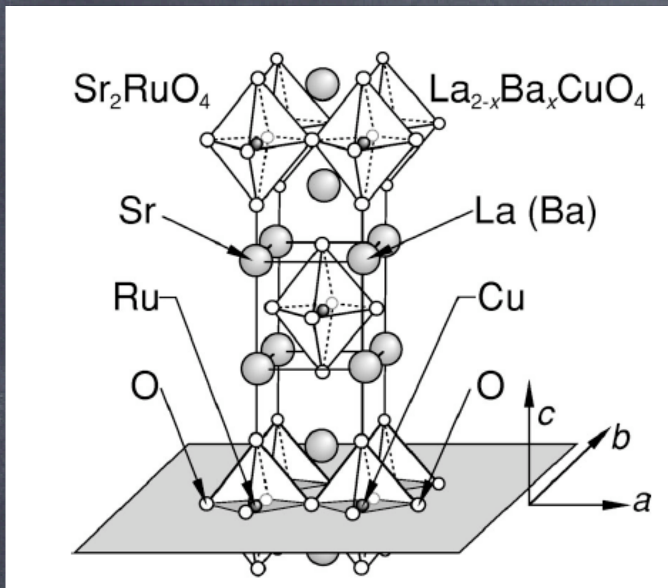
Sr_2RuO_4

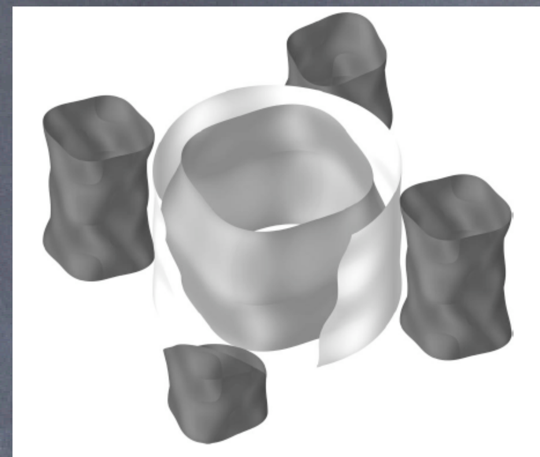




: P-T breaking SC

Sr_2RuO_4





: P-T breaking SC

- Gap function

$$\Delta_{ss'}(\mathbf{k}) = -\sum_{\mathbf{k}', s_3, s_4} V_{s'ss_3s_4}(\mathbf{k},\mathbf{k}')\langle a_{\mathbf{k}'s_3} a_{-\mathbf{k}'s_4}\rangle$$

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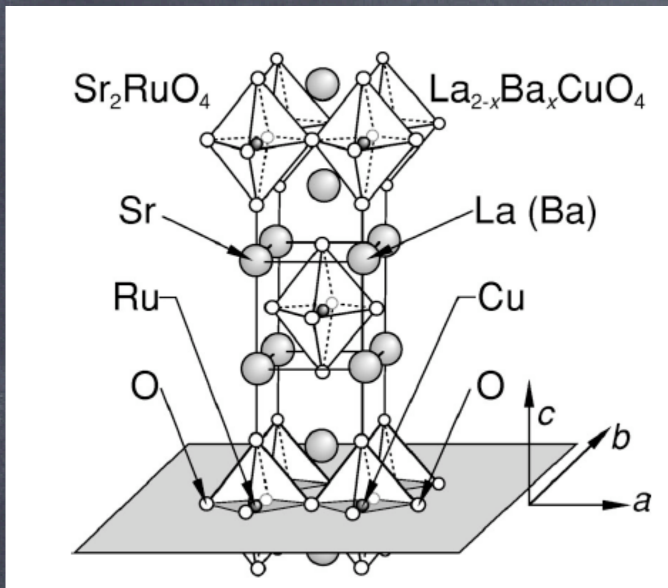

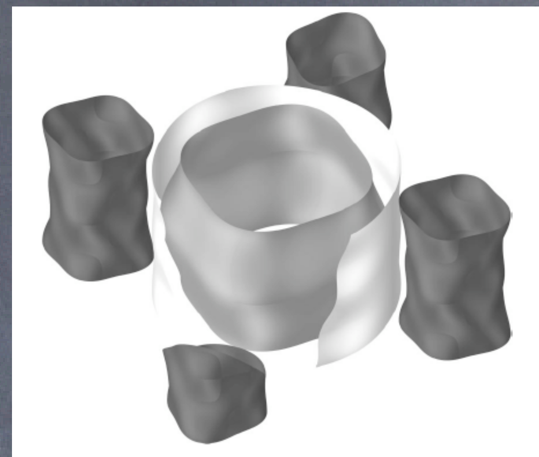

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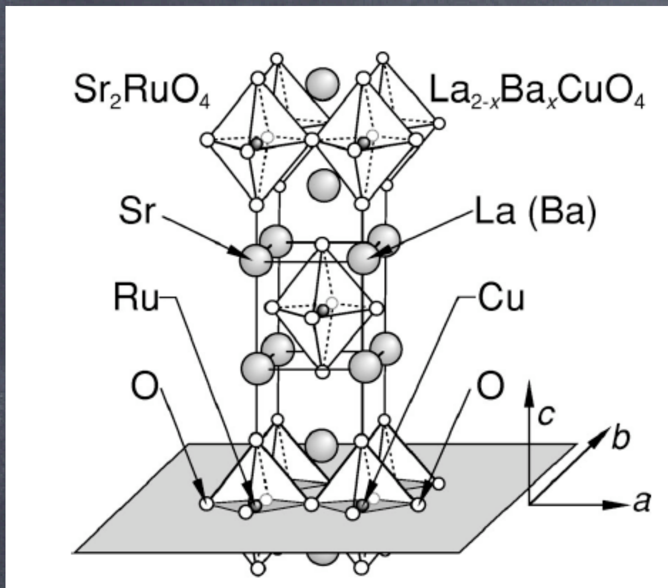

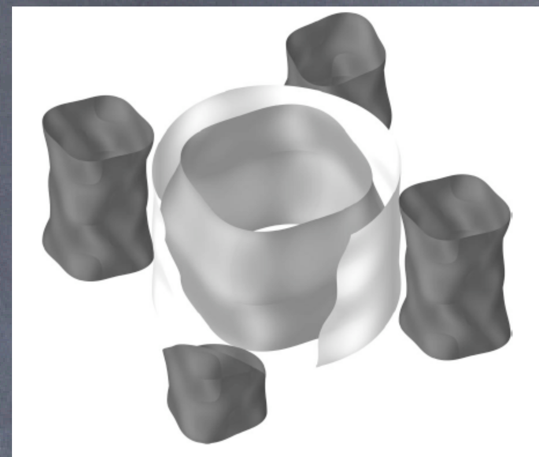

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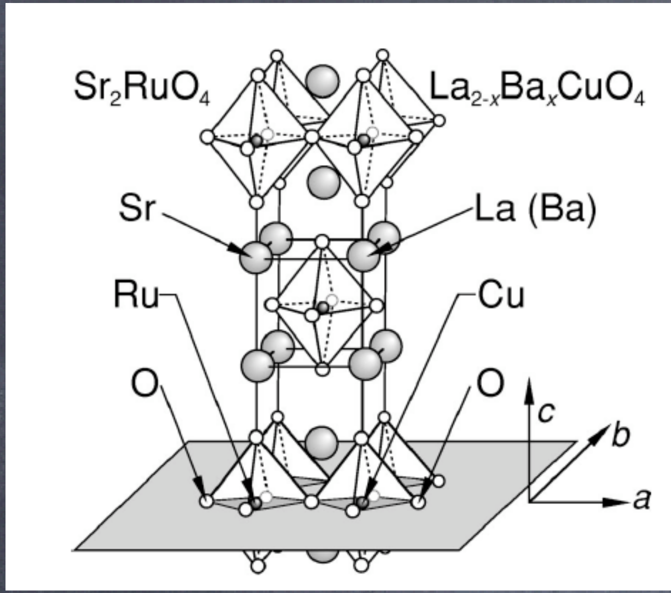

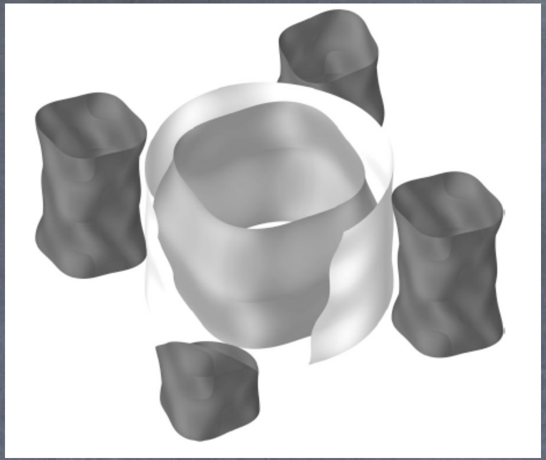

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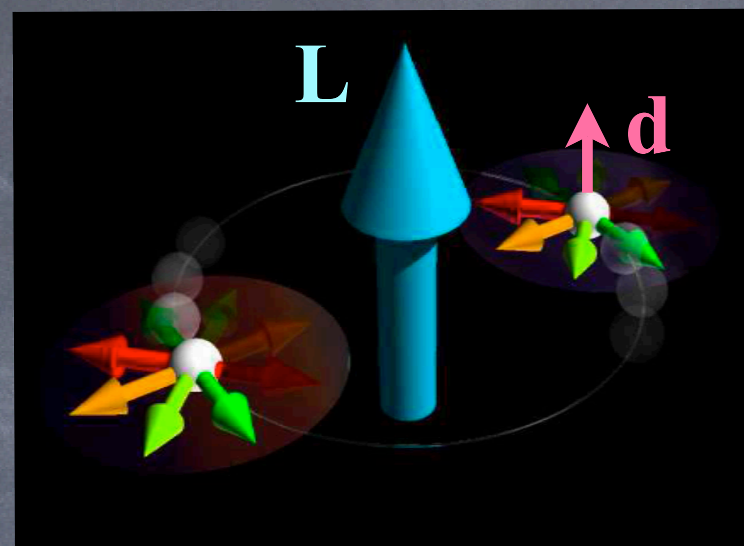
p+ip SC



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1/2 QV in p+ip SC

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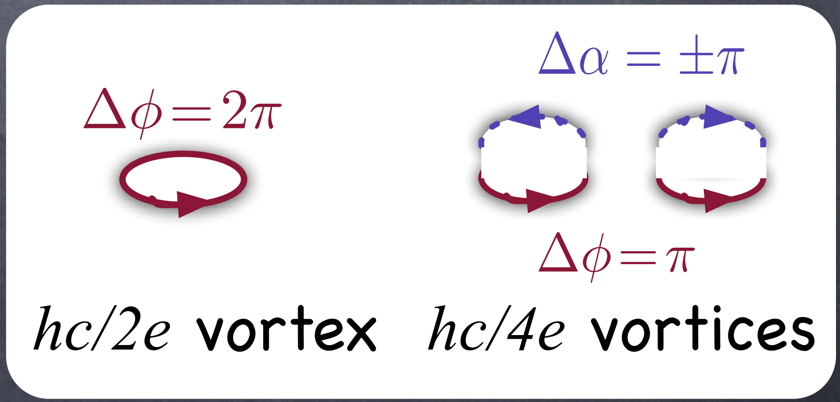
↔ 2π winding for only one spin component

↔ π winding of order parameter phase ϕ

+ π rotation of $\mathbf{d}$ vector



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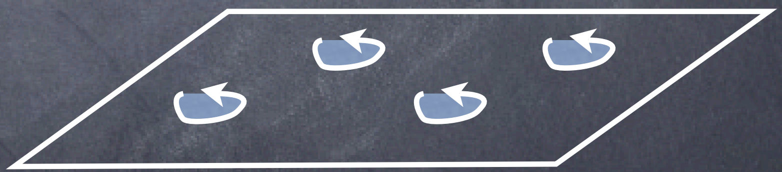

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Energetics

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$$f_{\rm grad}^{\rm 2D} = \frac{1}{2}\left(\frac{\hbar}{2m}\right)^2 \left[\rho_{\rm s}\left(\nabla_{\perp}\phi - \frac{2e}{\hbar c}\mathbf{A}\right)^2 + \rho_{\rm sp}\left(\nabla_{\perp}\alpha\right)^2\right] + \frac{1}{8\pi}(\nabla \times \mathbf{A})^2$$

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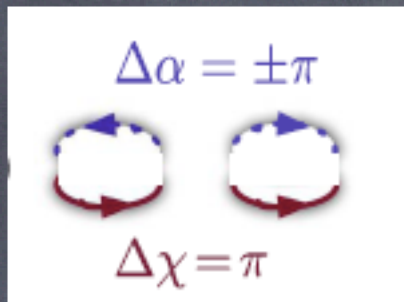
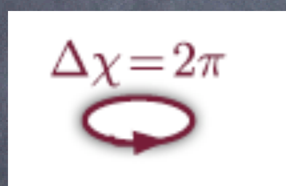
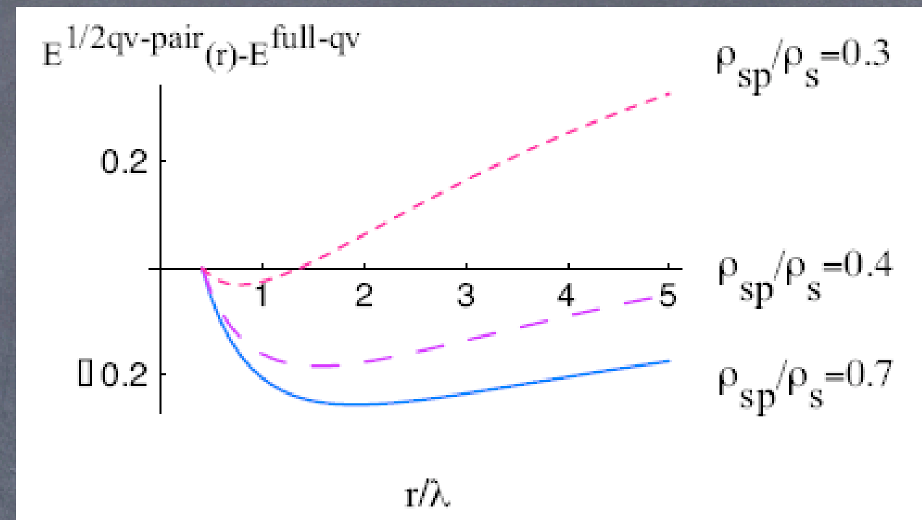
- Spin current energy diverges logarithmically!

stability of 1/2 QV







- Competition between screened magnetic repulsion and unscreened spin attraction
- Finite equilibrium size for small ρ_{sp}/ρ_s
- Use mesoscopic samples.

In search of topological states with fractionalized excitations.

- Now what?
  - How to prepare the states
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  - Where else?

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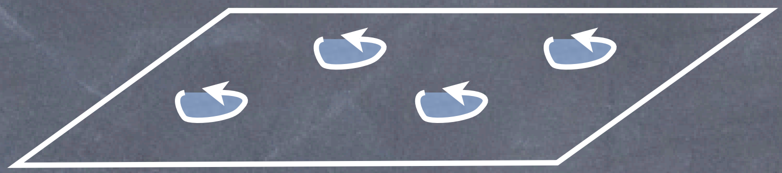

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References

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