Counting Read-Rezayi states in the thin torus limit (and in CFT)

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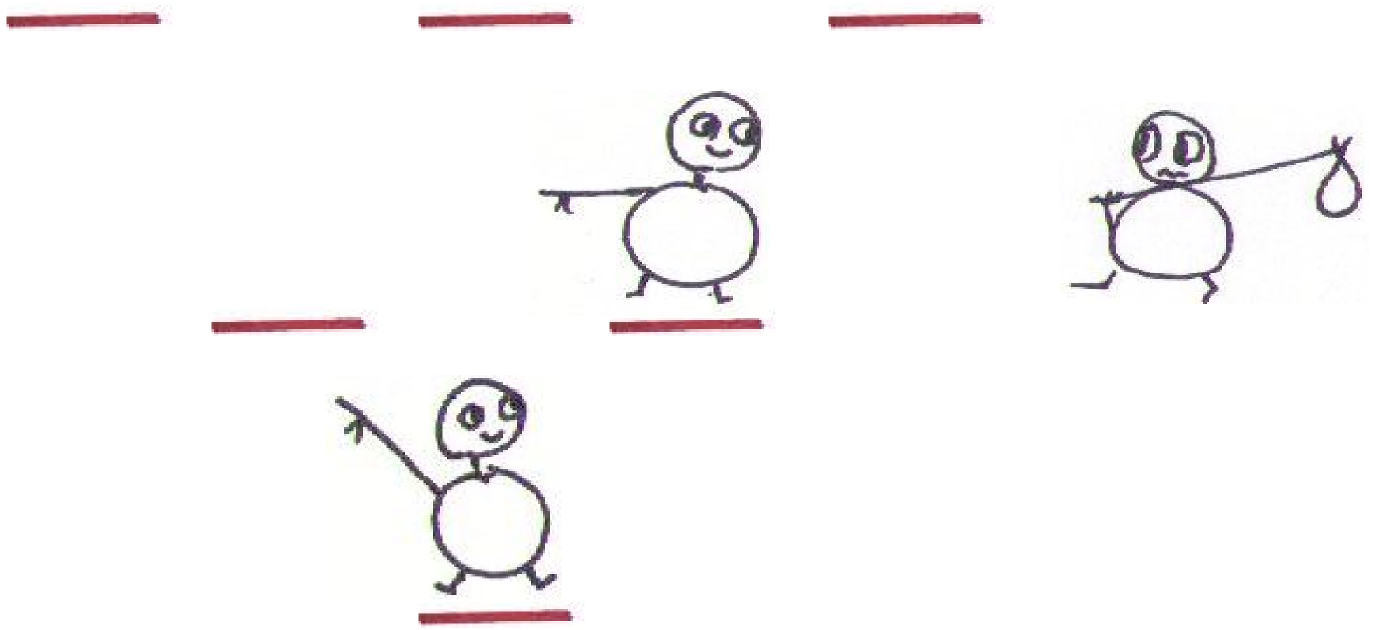
Papers: E. Ardonne, E. Bergholtz, J. K, and E. Wikberg,
J. Stat. Mech. P04016 (2008)
E. Bergholtz, J.K, E. Wikberg, H. Hansson and A. Karlhede,
PRB 74, 081308 (R) (2006)

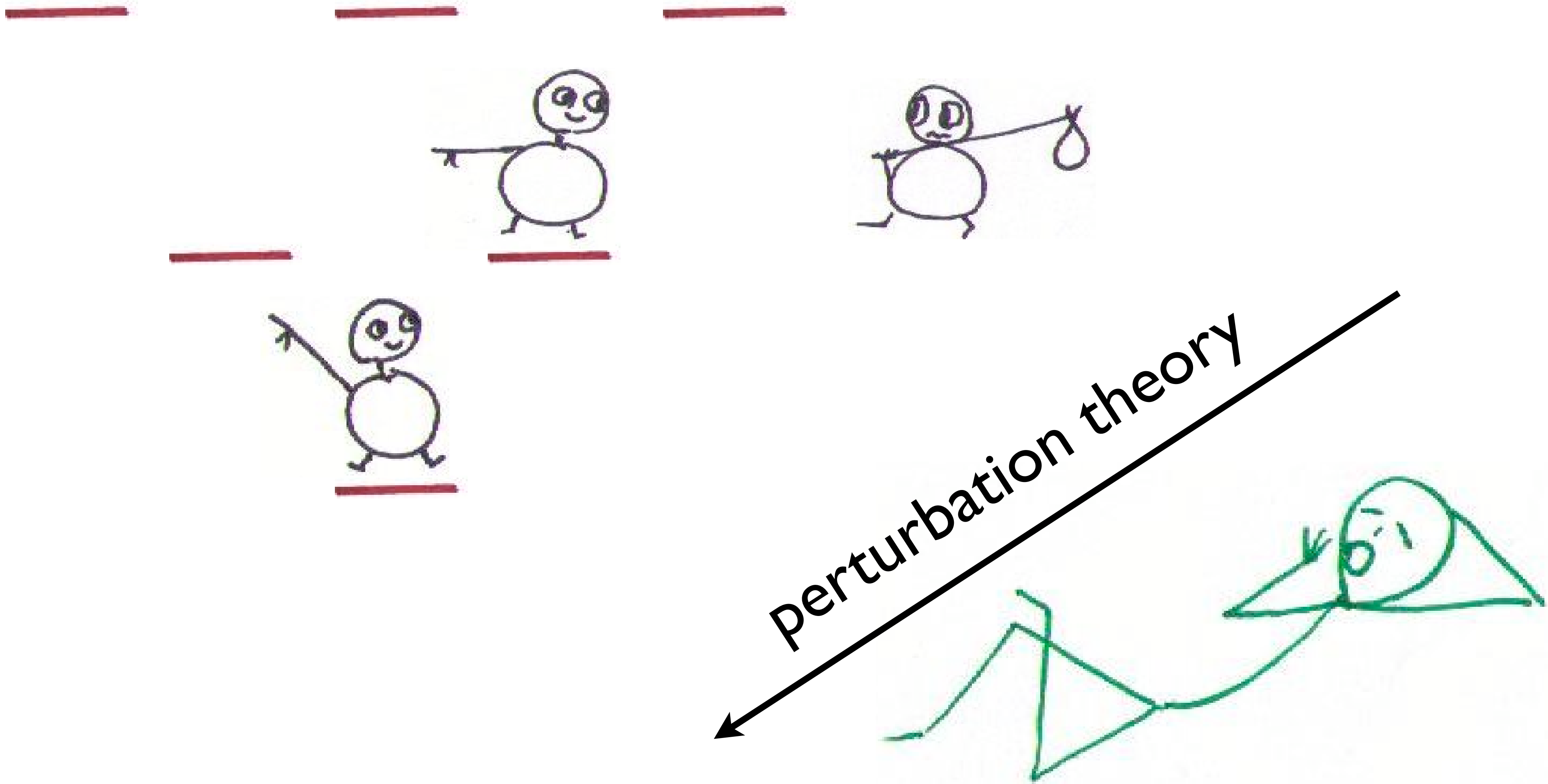
Outline

- The story in short
- The thin torus picture of the QHE
- Counting Read-Rezayi states.
- Short comparison with CFT
- Conclusions

Fermi gas

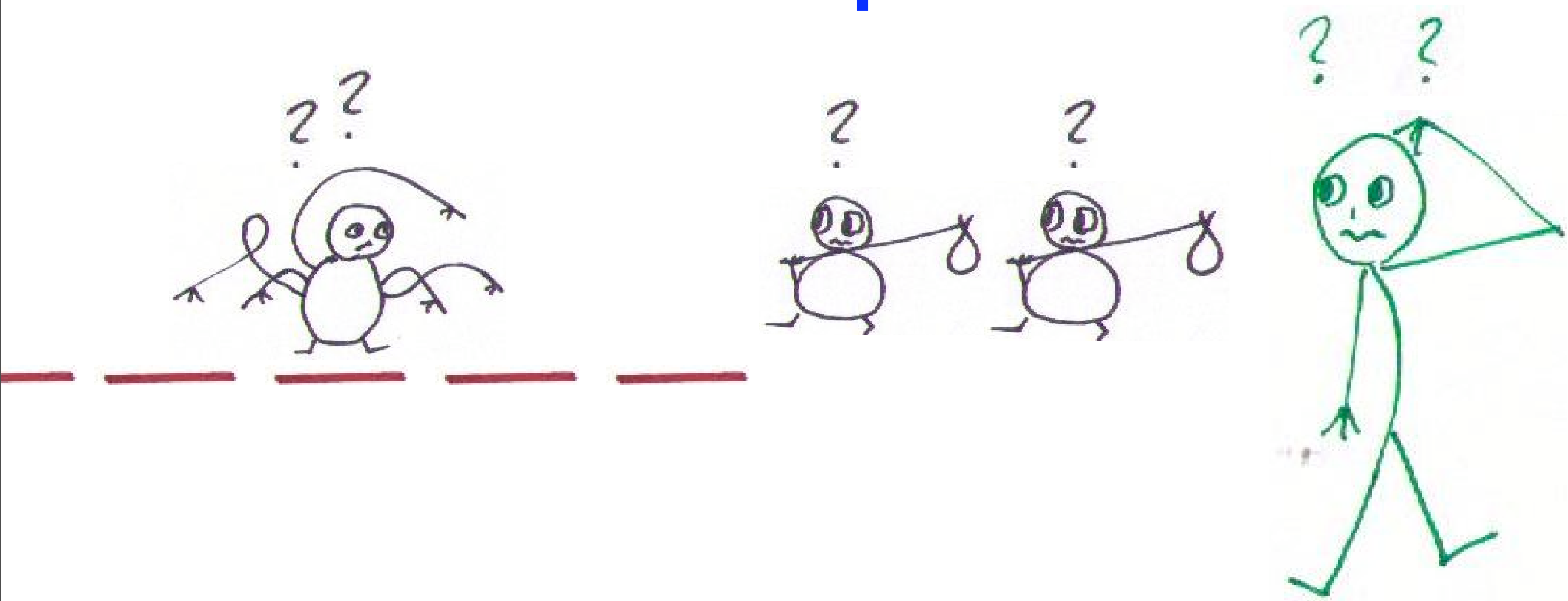


Fermi gas



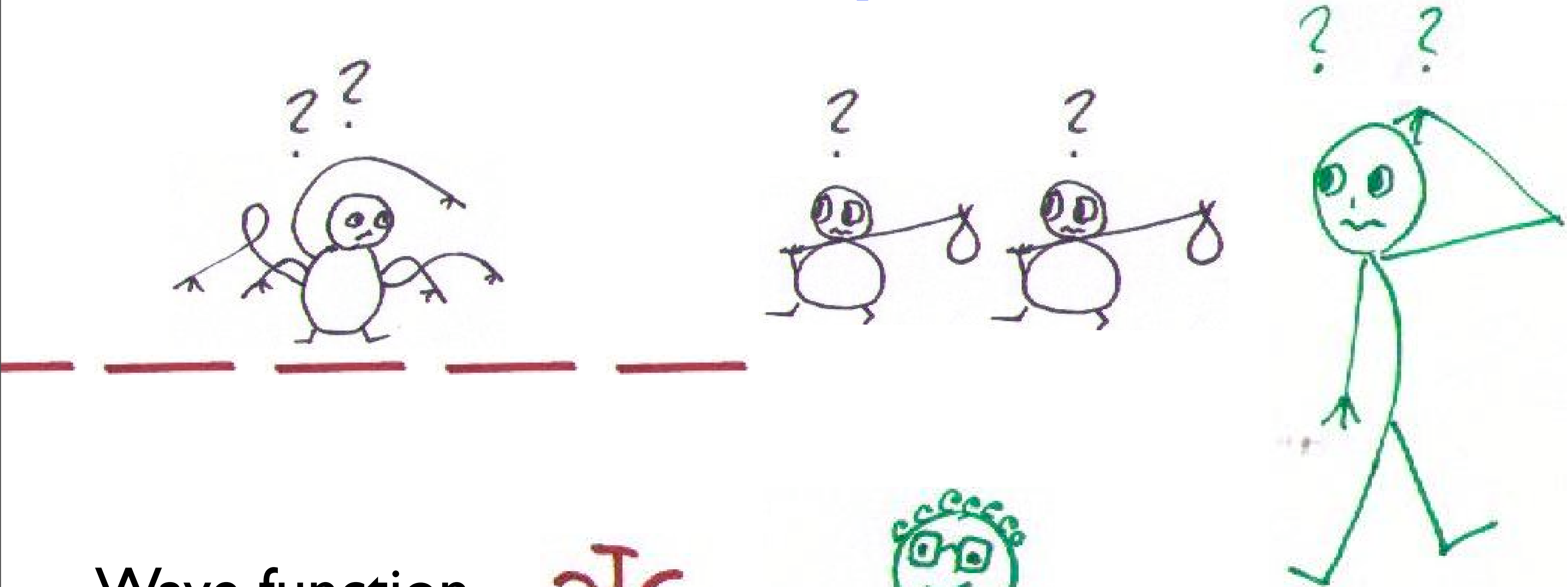
Fermi liquid

LLL liquid?



perturbation theory

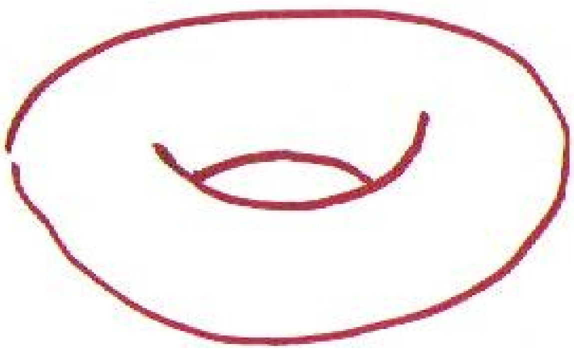
LLL liquid?



Wave function engineering

~~perturbation theory~~

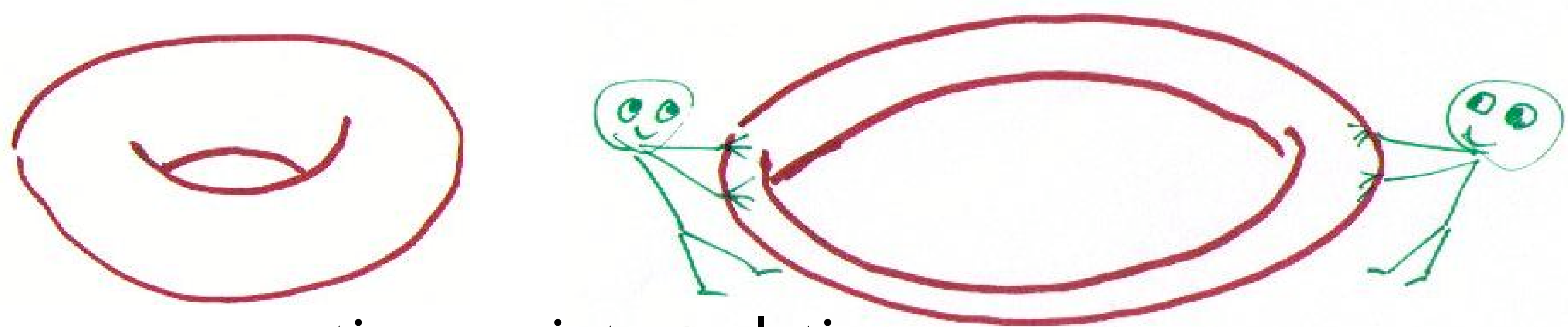
Topological liquid



Topological liquid

continuous interpolation

Topological liquid



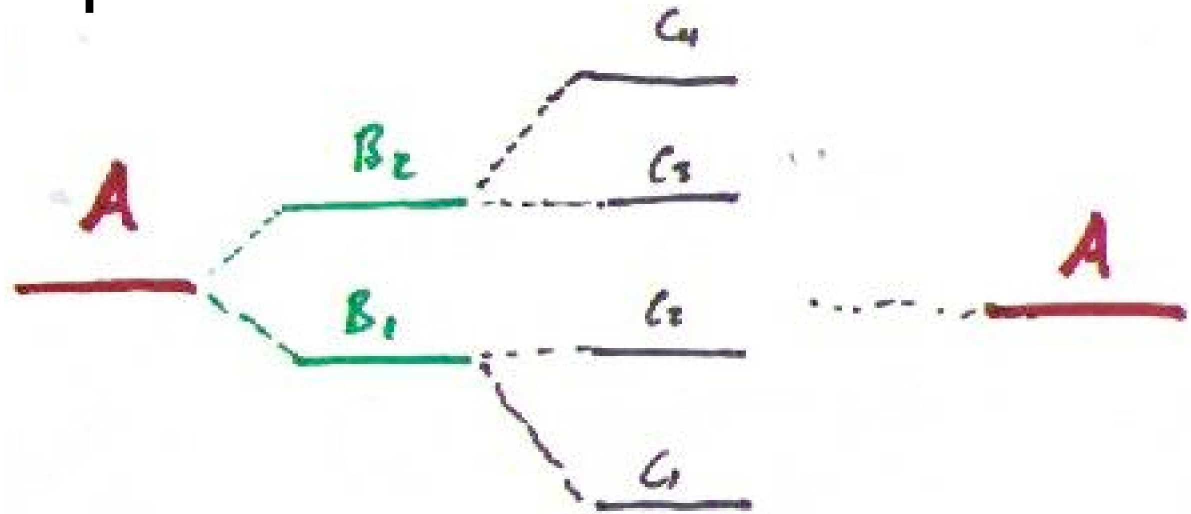
continuous interpolation

abelian states

Topological liquid

continuous interpolation

abelian states



nonabelian states

- Anderson (1983): TT + interaction = Laughlin states
- Su (1984): CDW on a small ν=1/3 system on the torus
- Haldane (1985): degeneracy on the torus
- Haldane & Rezayi (1994): ν=1/3 on a thin cylinder

The thin torus picture

- Bergholtz & Karlhede (PRL 2005, ...)
- Stockholm group, Berkeley, ... (papers 2006-)

Related: Haldane, Bernevig, ...: Jacks
Wen, ... : fusing electrons

1d lattice model

One particle states

$\ell = 1$

$$\psi_k \propto e^{-ikx} e^{-(y-k)^2/2}$$

$$k = \frac{2\pi n}{L_1}$$

1d lattice model

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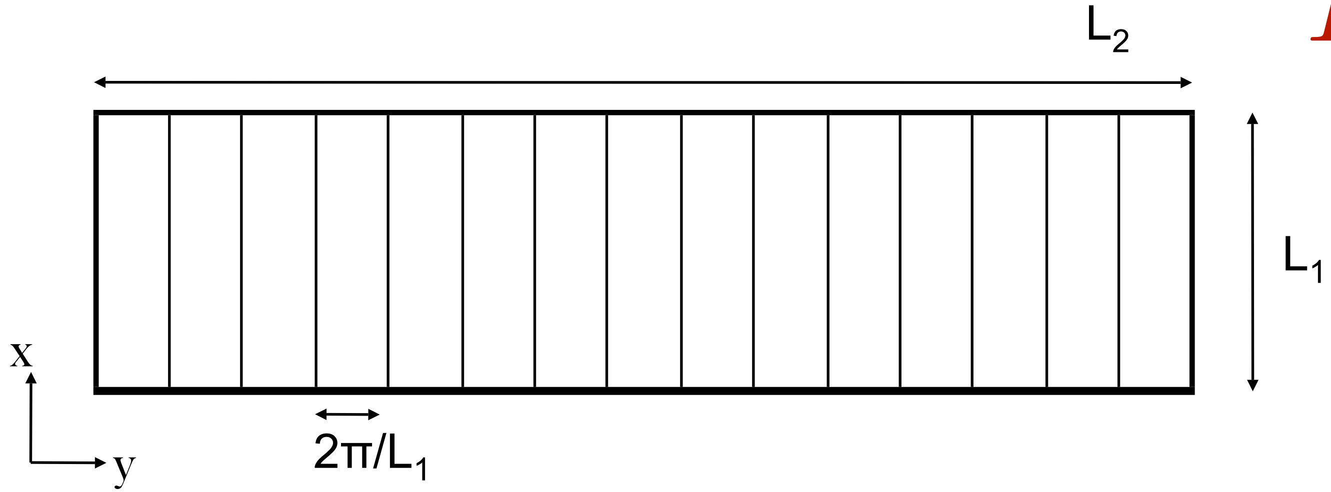

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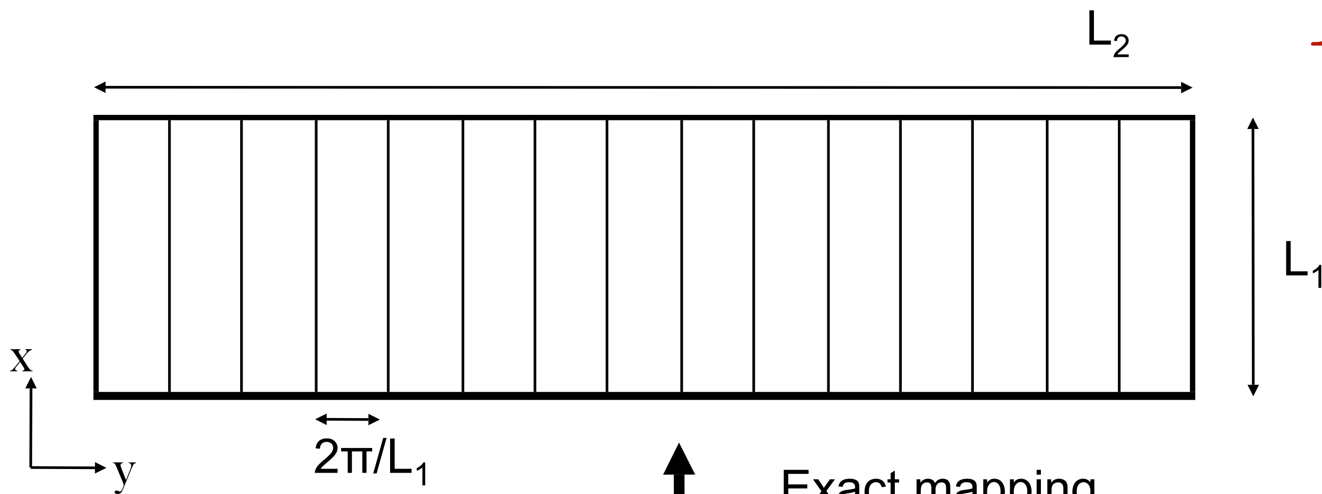

Exact mapping

Fock states
("1d basis" for
a 2d electron gas)

$|1\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ \rangle$

1d lattice model

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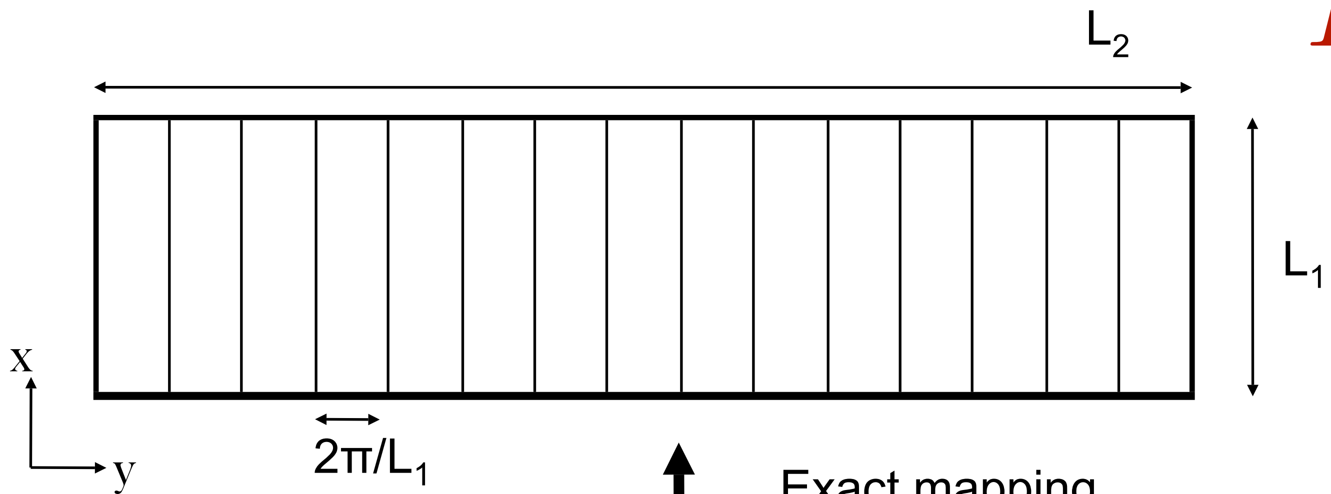
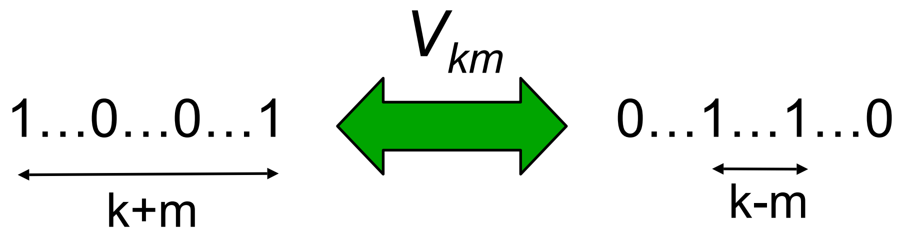


Exact mapping

Fock states
("1d basis" for
a 2d electron gas)

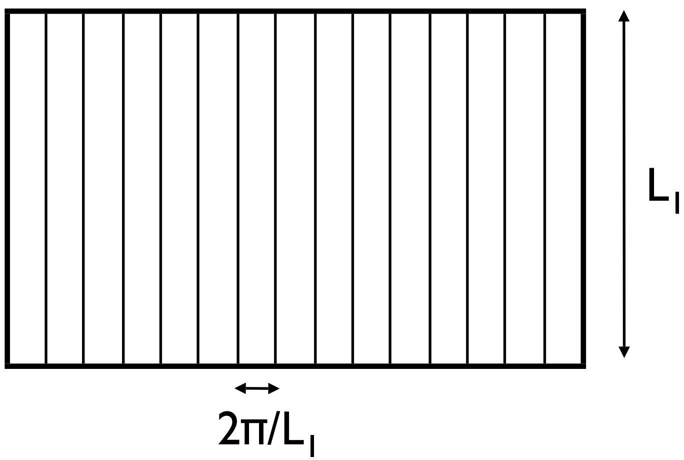
|1 0 0 1 0 0 1 0 0 0 1 1 0 0 0 >

Two-body
interction

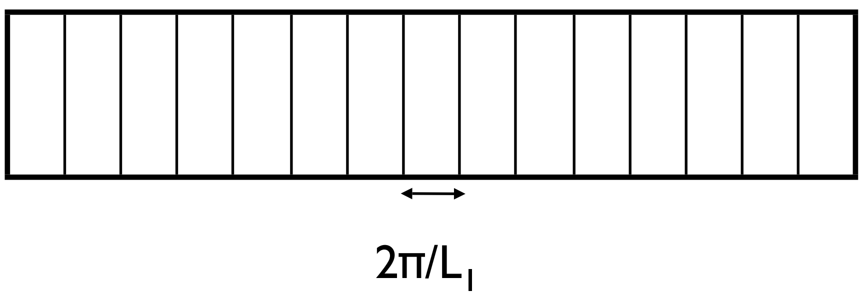


The thin torus

Large L_1, many ‘neighbor’ states



Small L_1, well separated states



- Experimental situation
- Complicated interaction

- Far from the experimental situation
- Simple model!

Continuous interpolation between the two cases!

- Quantum numbers the same
- Thin limit of bulk w.f.s
- CFT construction

- Numerical studies
- Special hyperlocal Hamiltonians

ν=1/3 on a cylinder

$$w_i = e^{i\frac{2\pi}{L_1}z_i}$$

$$\Psi_{\rm rel} \propto \prod_{i<j}(w_i - w_j)^3 =$$

$$= \mathcal{A}\left[\begin{array}{rl} - & (w_1)^0(w_2)^3(w_3)^6 \\ +3 & (w_1)^0(w_2)^4(w_3)^5 \\ +3 & (w_1)^1(w_2)^2(w_3)^6 \\ -6 & (w_1)^1(w_2)^3(w_3)^5 \\ +15 & (w_1)^2(w_2)^3(w_3)^4 \end{array}\right] \begin{array}{rl} \longrightarrow & 1001001 \\ \longrightarrow & 1000110 \\ \longrightarrow & 0110001 \\ \longrightarrow & 0101010 \\ \longrightarrow & 0011100 \end{array}$$

$$k = \frac{2\pi n}{L_1}$$

$$w^n \propto e^{y^2/2} \boxed{e^{k^2/2}} \psi_k$$

Thin torus ground states

(Tao-Thouless states)

$\nu = 1/3$ 100100100100…

$\nu = 2/5$ 10100101001 0…

Wave functions and electrostatics give the same thing

- The excitations are gapped since electrostatics gives a cost to all other states
- q-fold degeneracy for $\nu = p/q$ (for *q*)

100100100100… = [100]
010010010010… = [010]
001001001001…= [001]

q translated ground state sectors

Quasiparticles as domain walls

100100100100100100100100100100 $\nu = 1/3$

Remove three zeros from the GS and add one unit cell 100.

100101001001010010010010100100

[100] [001] [010] [100]

This creates three quasielectrons with charge -e/3!

Quasiparticles as domain walls

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Quasiparticles as domain walls

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001001001001...

100101001001010010010010100100

[100] [001] [010] [100]

This creates three quasielectrons with charge -e/3!

Two quasiholes. (Now $N_\phi \equiv 2 \pmod 3$ for PBC.)

10010001001001000100100

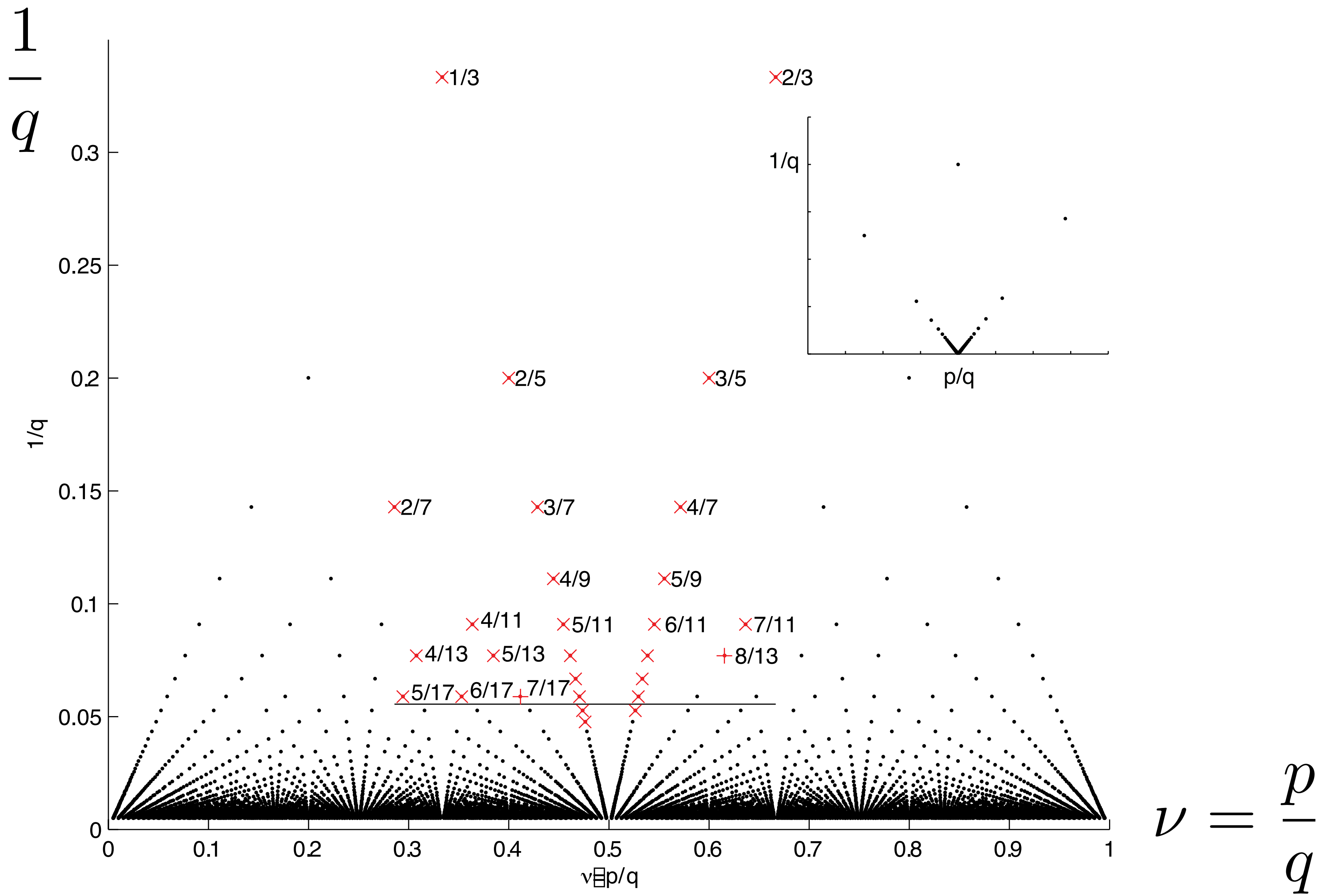
[100] [010] [001]

OBS! Degeneracy still 3!

Abelian states

- TT useful not only for topological quantum numbers: calculation of the gap (stability), comparison with experiments etc
- explicit hierarchy construction of general $\nu=p/q$
- solvable model also for gapless states states (e.g. $\nu=1/2$)

Stability of hierachy states



Read-Rezayi states and counting the degeneracy of quasiparticle states

Read-Rezayi (parafermion) states

(Read & Rezayi, PRB 360, 362 (1999))

- A different generalization compared to CF.
- Nonabelian quasiparticles!
- $\nu = k/(kM+2)$
  - Laughlin states *k*=1 and filling fraction 1/(*M*+2).
  - Moore-Read state *k*=2, *M*=1.
- *M* =odd fermionic, *M* =even bosonic (interesting for rotating BEC).
- Exact ground state of a (*k*+1)-body ineraction!

“(k+1)-interaction” in the thin limit

Ground states at $\nu = k/(kM+2)$

“exclusion rules”

- Two adjacent particles at least *M* sites apart.
- Any *kM*+2 sites contain exactly *k* particles.

These rules give the thin limit of RR w.f.’s

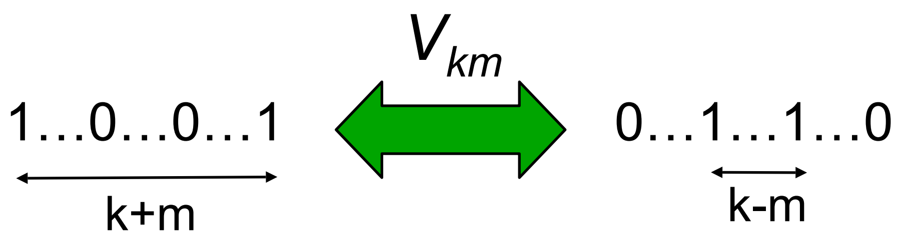
k=2, *M*=0

02020202= [02]
11111111= [11]
20202020= [20]

k=3, *M*=0

03030303= [03]
12121212= [12]
21212121= [21]
30303030= [30]

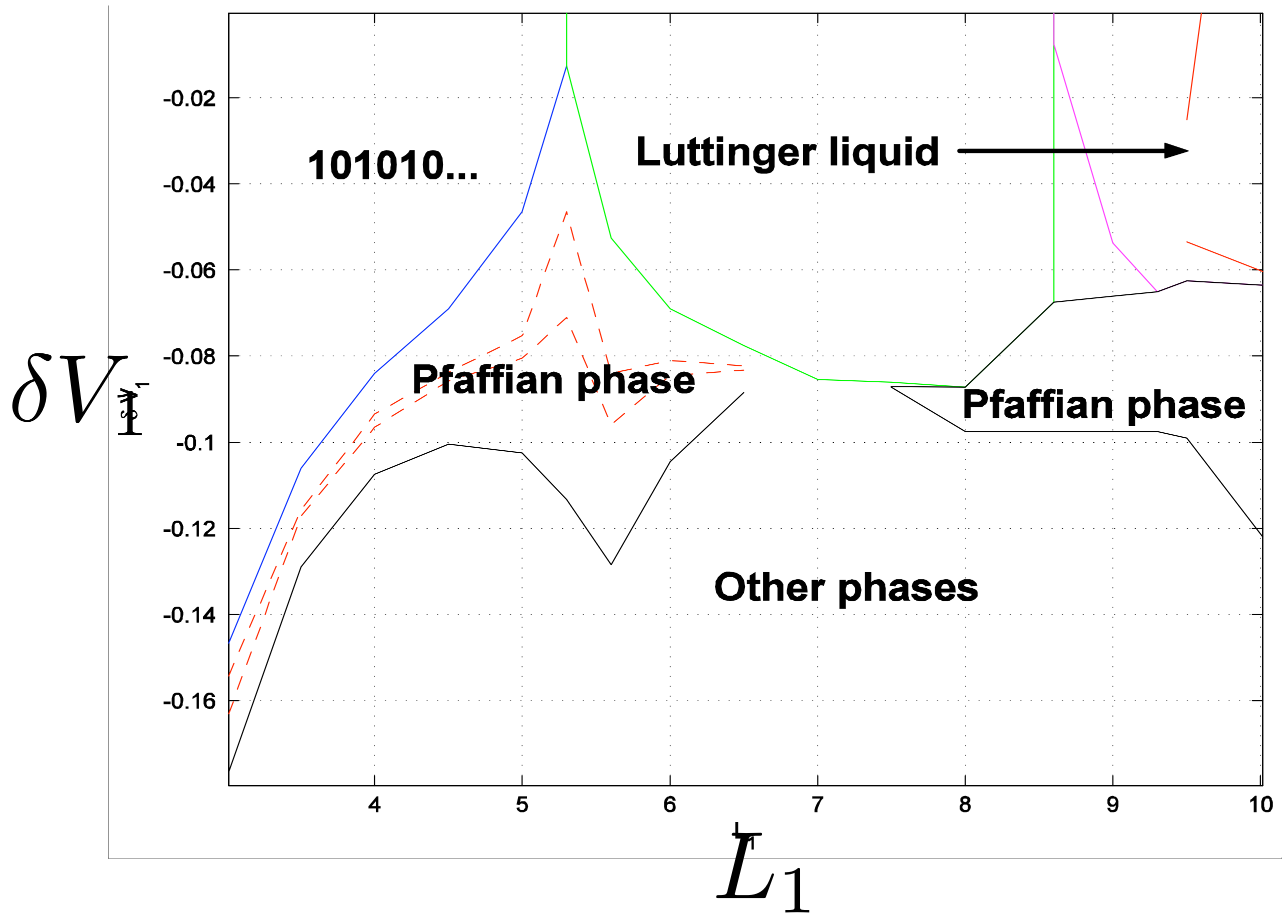
Physical interaction



00110011= [0011]
01010101= [0101]

- RR TT exactly degenerate for (k+1)-interaction
- but *not* for a *realistic* two-body interaction (in contrast to abelian states).
- Stability calculation not possible

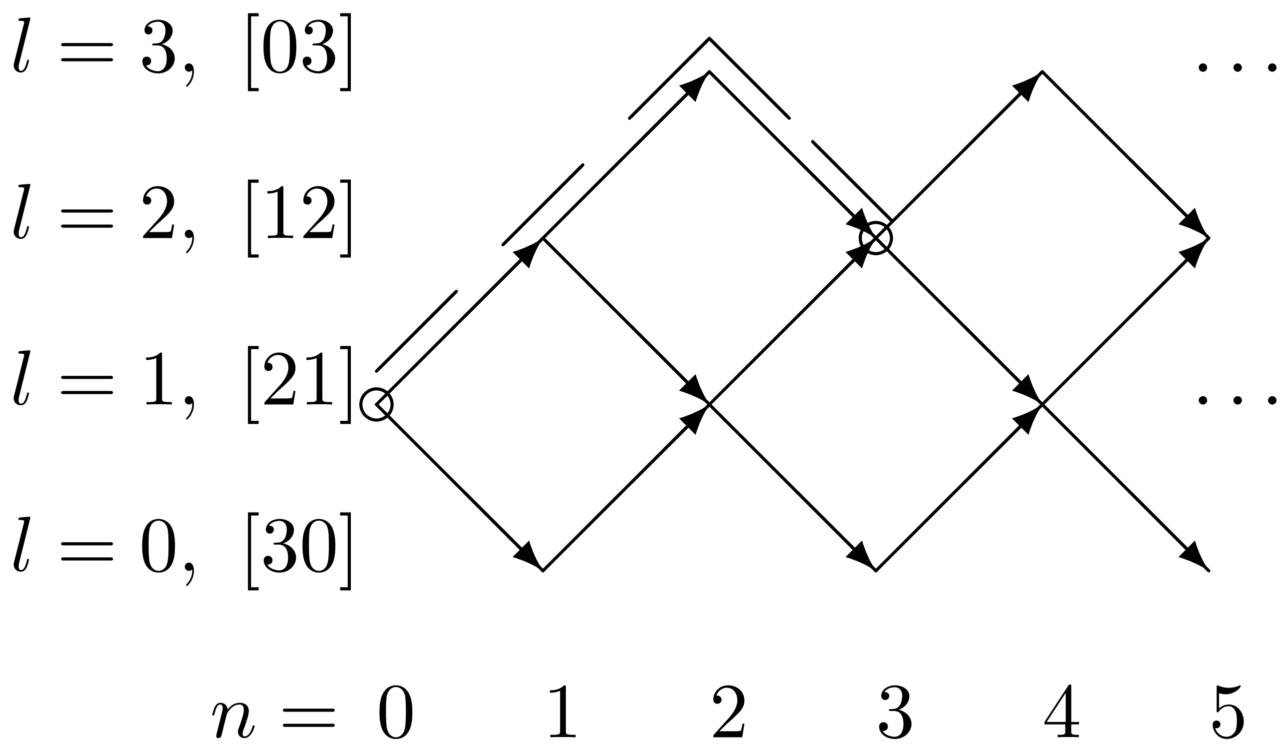
Two-body interaction (tuning pseudopotentials)



Quasiholes

2121121212030303021212 1

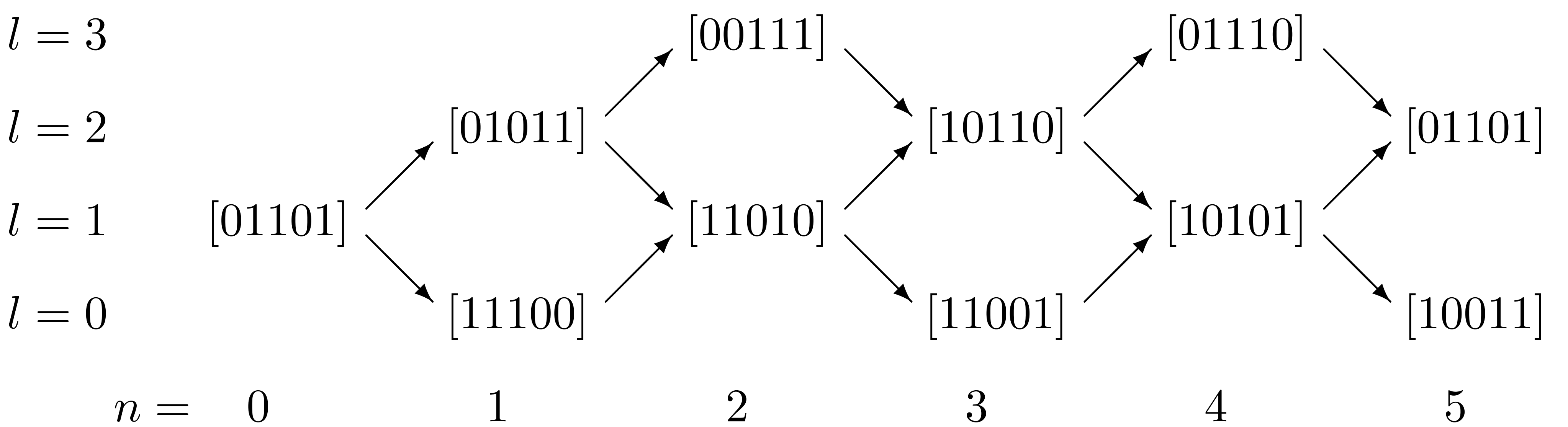
[21] [12] [03] [12]



$M > 0$?

M=0 2 1 2 1 1 2 1 2...

M=1 1101011010101101 0110...



M>0 contributes only with a overall translational degeneracy!

Degeneracy combinatorics

$$
\begin{array}{cc}
 & \begin{matrix} l = & 3 & 2 & 1 & 0 \end{matrix} \\
\begin{matrix} l = 3 \\ l = 2 \\ l = 1 \\ l = 0 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =: \mathbf{N}_1
\end{array}
$$

Total degeneracy on the torus for *n* domain walls

$$td(k, M, n, \delta) = \frac{kM + 2}{2} \mathrm{Tr}(\mathbf{N}_1^n \mathbf{B}^\delta)$$

Charge of quasiparticles

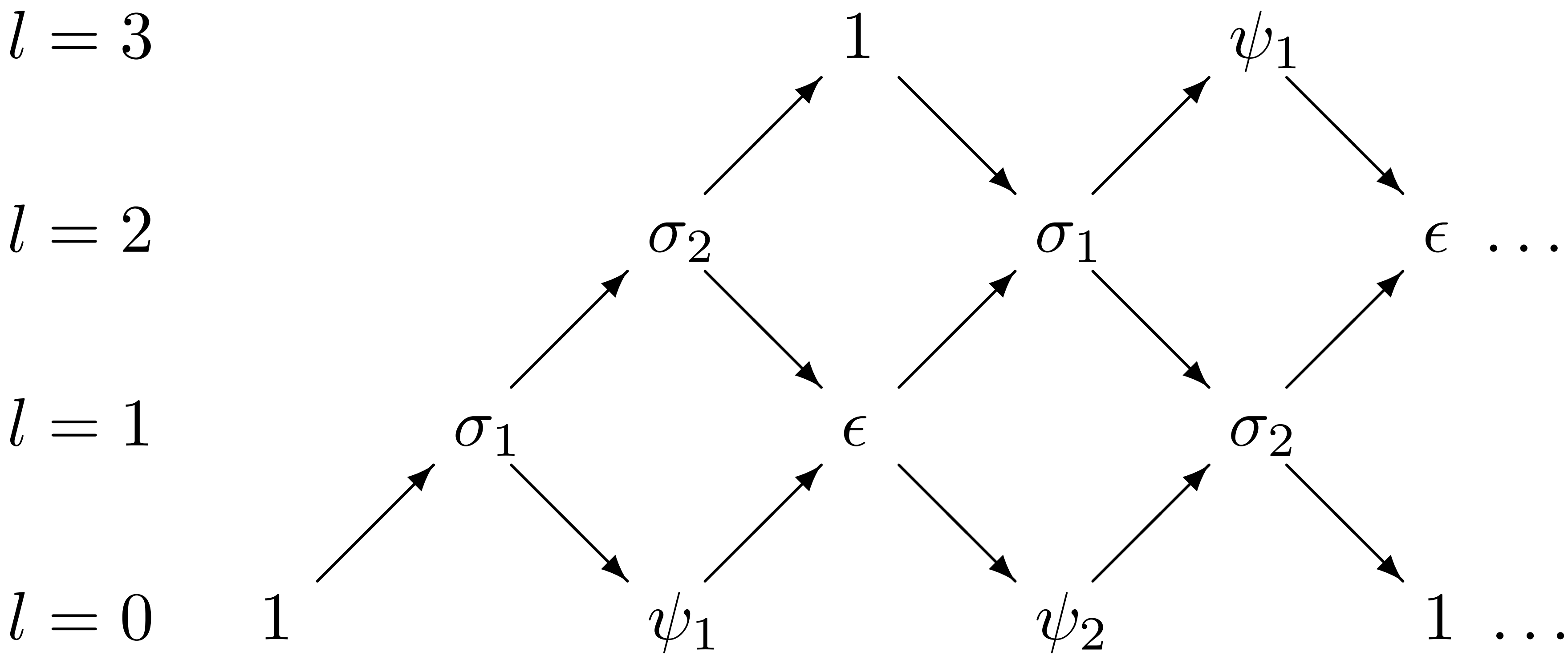
$$
\begin{aligned}
eN_e &= (N_\phi - n_{\mathrm{qp}} - n_{\mathrm{qh}})\frac{ek}{kM+2} + \\
&+ n_{\mathrm{qp}}\frac{e(k+1)}{kM+2} + n_{\mathrm{qh}}\frac{e(k-1)}{kM+2} = \\
&= N_\phi\frac{ek}{kM+2} + (n_{\mathrm{qp}} - n_{\mathrm{qh}})\boxed{\frac{e}{kM+2}}
\end{aligned}
$$

k=3
M=1

1101011010101101010110...

About the connection to the CFT calculation

$$\Phi^1_1 \times \Phi^l_m = \Phi^{l-1}_{m+1} + \Phi^{l+1}_{m+1}$$



Conclusions...

- Much of the QH physics is captured in the simple solvable thin torus limit.
- The domain wall sequences give an appealing representation of the fusion rules of nonabelian states.
- Statistics?