Conformal field theory approach to the abelian hierarchy

outline

- wave functions as CFT correlators
- Berry phase vs. monodromy
- quasi-electron operator
- Jain states and the hierarchy
- quasi-electron in the Moore-Read state

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Laughlin's wave functions "look like" correlation functions in a Conformal Field Theory.

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- generalization to other FQH states (Jain, Moore-Read, *etc*.)

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$\langle V_\alpha(z)V_\beta(w)\rangle \sim (z-w)^{\alpha\beta}$ $\longrightarrow$ fermionic statistics for α^2 odd integer

Laughlin's fractions: $\nu = 1/m$

Electron operator: $V_e(z) = e^{i\sqrt{m}\varphi(z)}$, $m = 2p + 1$

Quasihole operator: $V_h(\eta) = e^{\frac{i}{\sqrt{m}}\varphi(\eta)}$

charged particles with conserved current $J(z) = \frac{1}{\sqrt{m}}\partial_z\varphi(z)$

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can describe ground state

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and quasihole excitations

$$\Psi_L^{1qh}(\eta) = \langle V_h(\eta)\prod_{i=1}^{N} V_e(z_i)\mathcal{O}_{bg}\rangle$$

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Hierarchical wave functions at level n:

n electron operators:

recursive construction

$$V_1(z) =: e^{i\gamma_1\varphi_1(z)} : \qquad \gamma_1 = \sqrt{t_1}, \quad t_1 = 3, 5, \dots \, .$$

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$$\nu_n = \cfrac{1}{t_1 - \cfrac{1}{t_2 - \cfrac{1}{\ddots \, \cfrac{}{t_{n-1} - \cfrac{1}{t_n}}}}}$$

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- **Identical to Jain wave functions whenever these exist**
- **Reduce to exact ground state in the thin torus limit**

Monodromy vs. Berry's phase

Laughlin fractions

- statistical phase = monodromy + Berry phase
- wave functions not normalized

$$\Psi_{1/3}^{2qh} = \mathcal{N}(\eta_1, \bar{\eta}_1, \eta_2, \bar{\eta}_2) \prod_{i,a} (z_i - \eta_a) \prod_{i<j} (z_i - z_j)^3$$

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free interpolation between different monodromies by introducing a second uncharged bosonic field

$$\begin{aligned} H(\eta) &= e^{\frac{i}{\sqrt{3}}\varphi_1(\eta)} \\ H'(\eta) &= e^{\frac{i}{\sqrt{3}}\varphi_1(\eta) + i\alpha\varphi_2(\eta)} \end{aligned}$$

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- $\alpha = \frac{5}{\sqrt{15}}$ bosonic representation
- $\alpha = \frac{2}{\sqrt{15}}$: $\Psi_{1/3}^{2qh} = \mathcal{N}''(\eta_1, \bar{\eta}_1, \eta_2, \bar{\eta}_2) \cdot (\eta_1 - \eta_2)^{1/5} \prod_{i,a} (z_i - \eta_a) \prod_{i<j} (z_i - z_j)^3$ **valid representation**

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Laughlin's Quasielectron:

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is not without trouble........

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Jain's Quasielectron:

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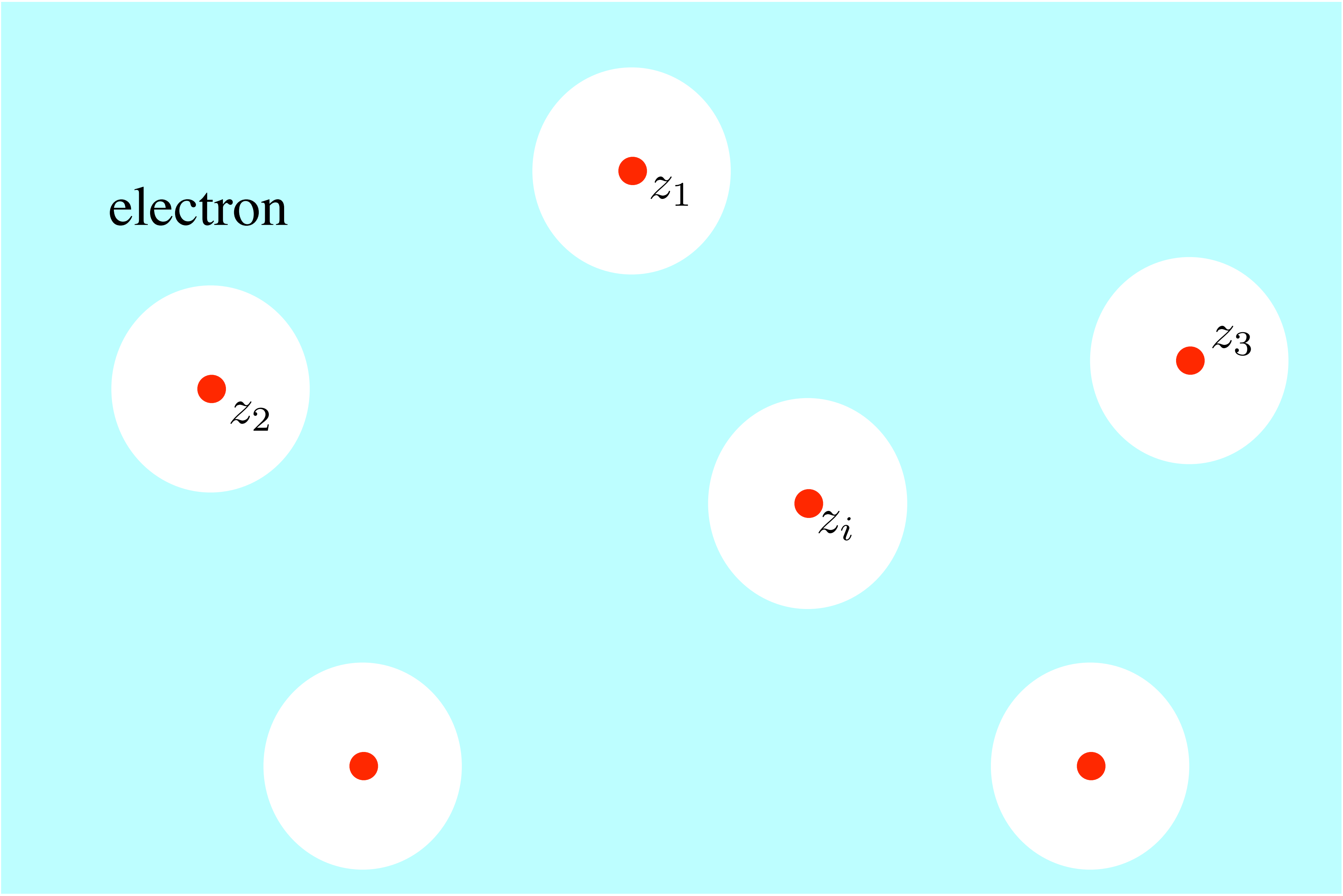

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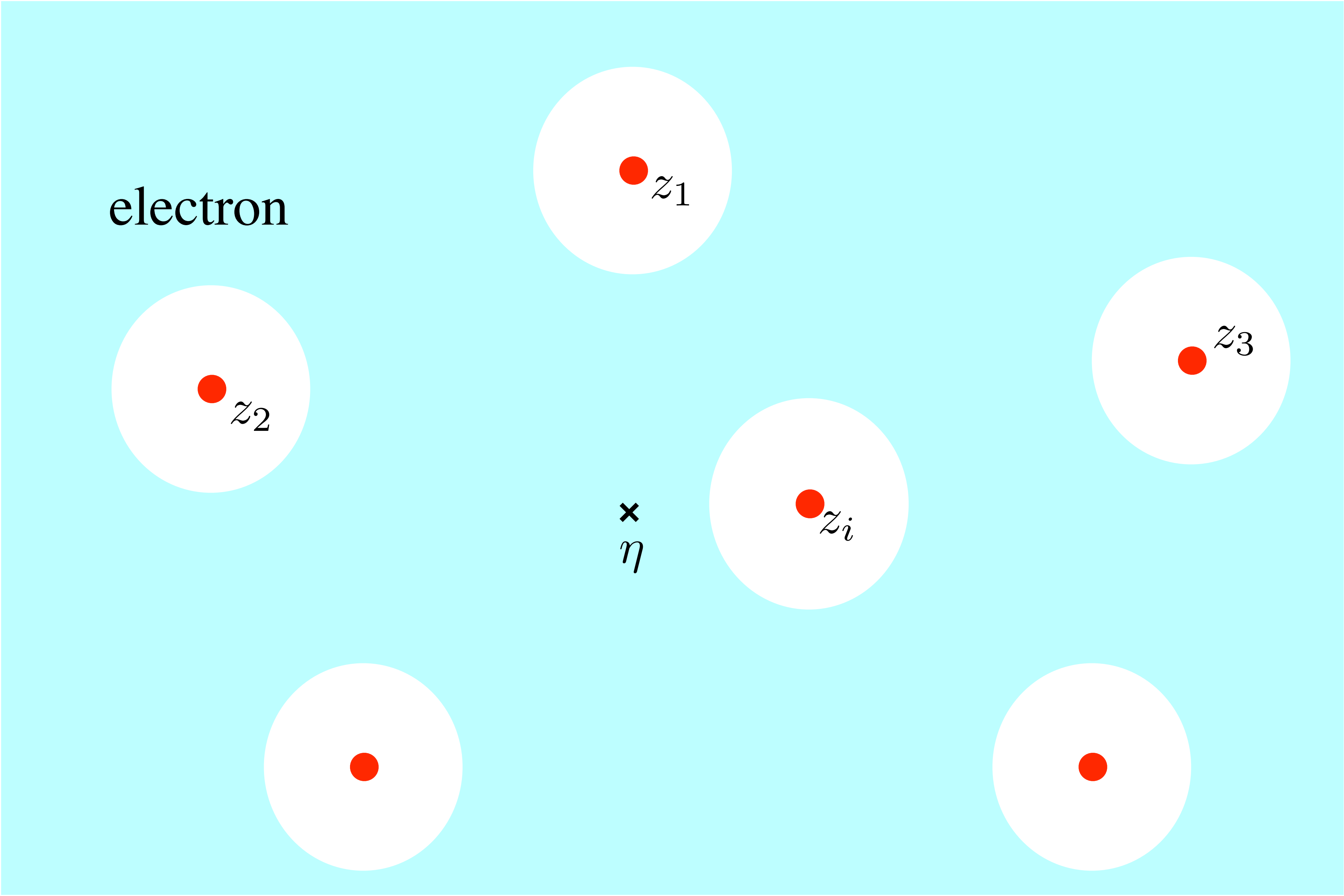

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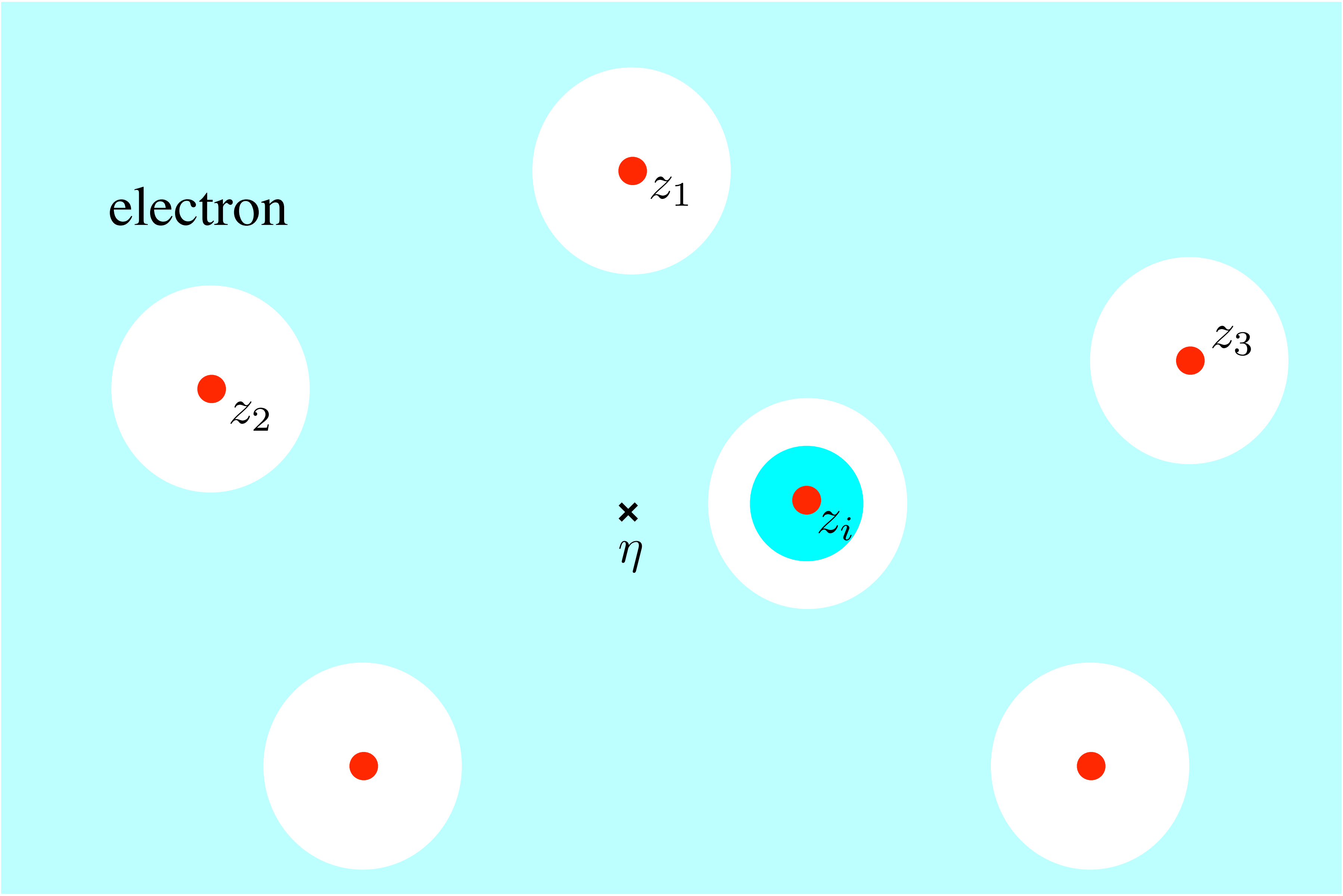

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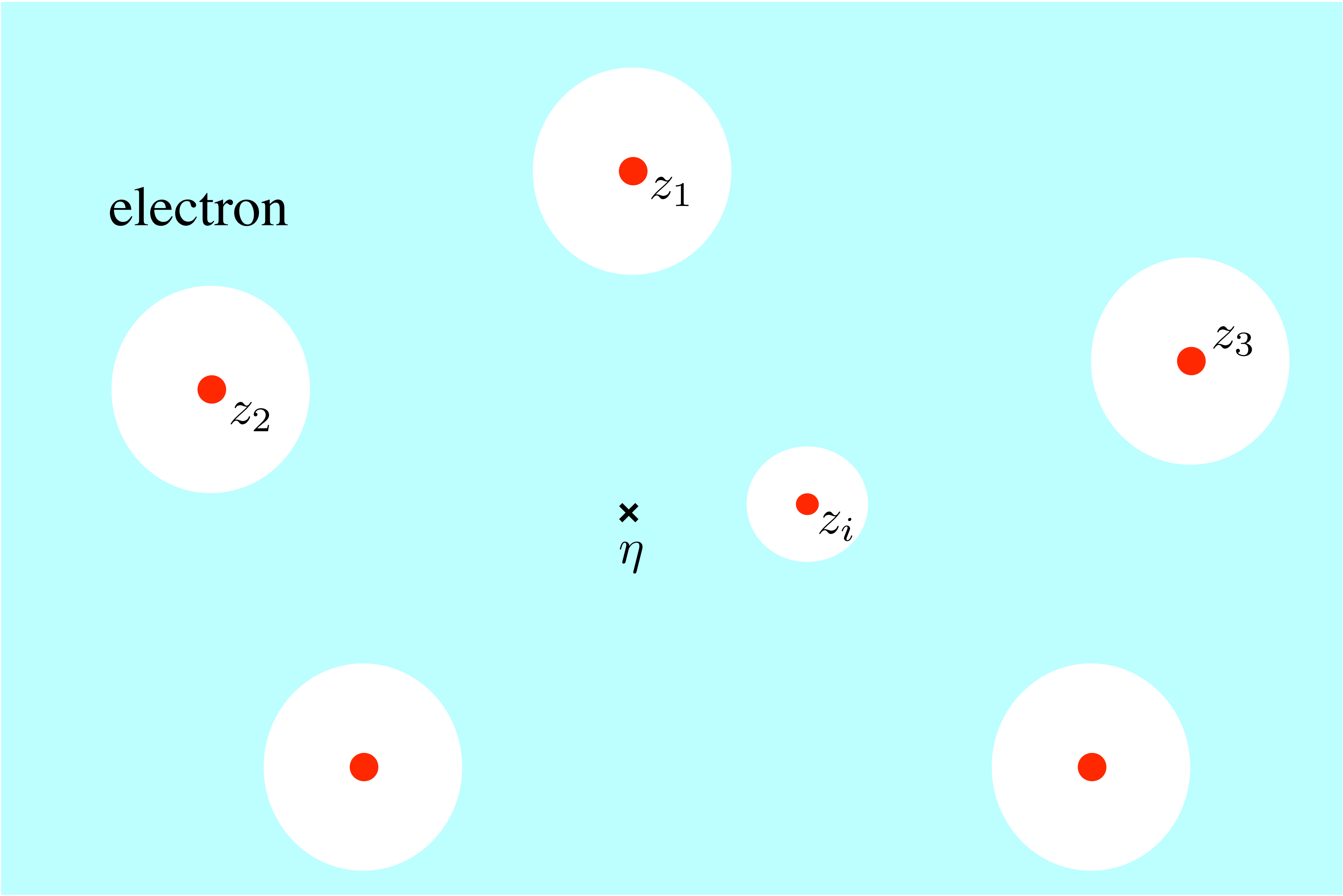

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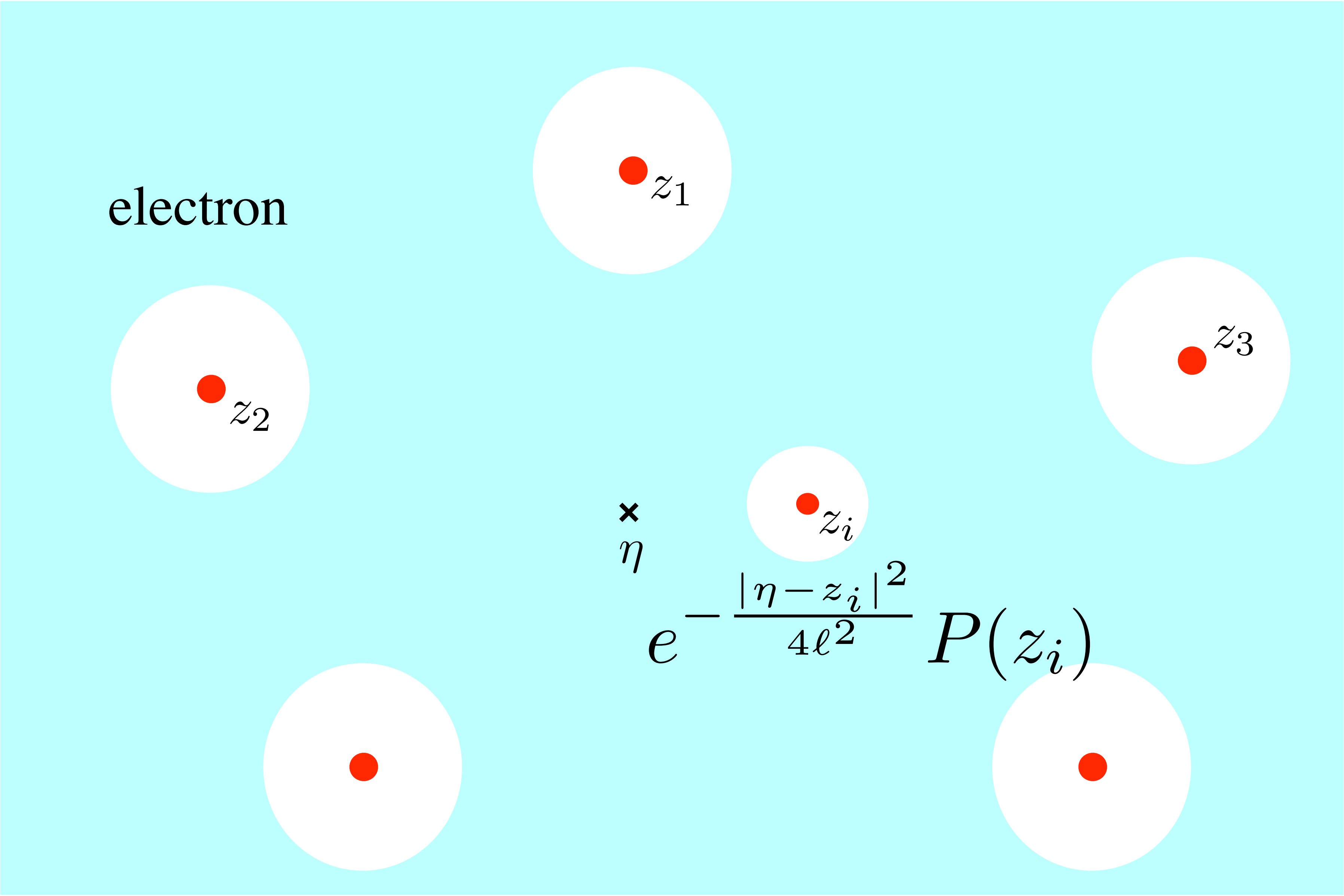

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$$= \int d^2w \; e^{-\frac{1}{4m}(|w|^2 - 2w\bar{\eta} + |\eta|^2)} \quad \bar{\partial}_w J(w) H^{-1}(w)$$

The quasielectron operator

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1. fix statistics by hand

2. inverse hole with bosonic statistics $\left(H^{(b)}\right)^{-1}(\eta) = e^{-i\frac{1}{\sqrt{m}}\varphi_1(\eta)} e^{i\frac{\sqrt{2m-1}}{\sqrt{m}}\varphi_2(\eta)}$

The quasielectron operator

$$\mathcal{P}(\vec{R}) = e^{-\frac{|\eta|^2}{4m}}\mathcal{P}(\eta)$$
$$= \int d^2w\; e^{-\frac{1}{4m}(|w|^2-2w\bar{\eta}+|\eta|^2)}\left(\bar{\partial}_w J(w)\left(H^{(b)}\right)^{-1}(w)\right)$$

Support only at the electron positions z_i

'gaussian' weight

bosonic

$$\Psi^{1qe} = \langle \mathcal{P}(\vec{R})\prod_{i=1}^{N}V(z_i)\mathcal{O}_{back}\rangle$$

reproduces Jain's quasielectron up to boundary terms

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Comments:

- possible to choose fermionic statistics, gives similar wave functions but not identical
- for hierarchical states at higher level: **(...)** more complicated

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Results:

- **quasi-local on magnetic length scale**
- **same charge and conformal dimension as $H^{-1}(\eta)$, but bosonic statistics**
- **construction completely in the lowest LL (no need for projection)**
- **multiple insertion gives multi-quasielectron states**

Jain states and the hierarchy

RESULT: Jain's wave function at ν=2/5 can be written as a quasielectron condensate of the Laughlin state at ν=1/3

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- *k* determines quasiparticle density
- *M*/*N* is fixed by homogeneity
- exponential factors combine to a lowest LL δ- function $\rightarrow$ integrals become trivial

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- k=0 gives Jain wave function at ν=2/5
- k=1 gives trial wave function for ν=4/11
- higher values for k reproduce whole series of quasiparticle condensates at lower density/filling fraction, identical to wave functions shown earlier

Quasielectrons in the Moore-Read state (ν=5/2)

Ising representation

$$\begin{aligned} \psi \times \psi &= \mathbf{1} \\ \sigma \times \psi &= \sigma \\ \sigma \times \sigma &= \mathbf{1} + \psi \end{aligned}$$	$$\begin{aligned} V(z) &= \psi(z) e^{i\sqrt{2}\varphi(z)} \\ H(\eta) &= \sigma(\eta) e^{\frac{i}{2\sqrt{2}}\varphi(\eta)} \end{aligned}$$

generalization of our construction to non-abelian quasiparticle:

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however, not without problems:

$$\Psi^{4qe} = \langle \mathcal{P}(\eta_1) \ldots \mathcal{P}(\eta_4) \prod_{i=1}^{N} V(z_i) \rangle \text{ contains terms as } eg. \ \langle \sigma(z_1) \ldots \sigma(z_4) \prod_{j=5}^{N} \psi(z_j) \rangle$$

wave functions are not analytic in electron coordinates

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similar problem as in abelian case → similar solutions:

- Replace non-analytic factors by hand
- 'Bosonize' the quasielectron operator

Monodromy vs. Berry's phase, the second

Ising representation

$$
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\end{aligned}
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$$\Psi_{(12)(34)} = \mathrm{PF}\left(\frac{(\eta_1-z_i)(\eta_2-z_i)(\eta_3-z_j)(\eta_4-z_j)+(i\leftrightarrow j)}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2 \quad , \quad x = \frac{(\eta_1-\eta_2)(\eta_3-\eta_4)}{(\eta_1-\eta_3)(\eta_2-\eta_4)}$$

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change of basis →

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basis states are given by different orderings of H_+ and H_-

$$\Psi_{(13)(24)} \sim \langle H_+(\eta_1)H_-(\eta_2)H_+(\eta_3)H_-(\eta_4)\prod_{j=1}^{N}V(z_j)\rangle$$

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And bosonize the quasihole:

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Different orderings of '+' and '-' should span the 2D Hilbert space for 4 quasiparticle excitation

$$\langle \mathcal{P}_+(\eta_1)\mathcal{P}_+(\eta_2)\mathcal{P}_-(\eta_3)\mathcal{P}_-(\eta_4) V(z_1)\ldots V(z_N)\rangle = \Psi_{(\eta_1,\eta_2)(\eta_3,\eta_4)}(z_1\ldots z_N)$$
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- **wave function does not correspond to a specific fusion channel of the quasiparticles**
- **non-abelian statistics is hidden entirely in the Berry phase**

Conclusions

- Ground state wave functions of hierarchical states (that are condensates of quasielectrons only) as CFT correlators
- Description of quasihole and quasielectron excitations in these states
- (Quasi-)local quasielectron operator
- Explicit connection between the Jain states and the hierarchy
- Generalization and outlook on non-abelian quasiparticle excitations