

# Measurement-Only Topological Quantum Computation

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Microsoft Station Q

Nordita Conference - Conformal Field  
Theory Approach to Quantum Hall Physics

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work done in collaboration with:

Mike Freedman and Chetan Nayak

arXiv:0802.0279 (PRL '08) and arXiv:0808.1933

# Introduction

- Non-Abelian anyons probably exist in certain gapped two dimensional systems:
  - Fractional Quantum Hall Effect ( $\nu=5/2, 12/5, \dots?$ )
  - ruthenates, topological insulators, rapidly rotating bose condensates, quantum loop gases/string nets?
- They could have application in quantum computation, providing naturally (“topologically protected”) fault-tolerant hardware.
- Assuming we have them at our disposal, what operations are necessary to implement topological quantum computation?

# Anyon Models

(unitary braided tensor categories)

Describe quasiparticle braiding statistics in gapped two dimensional systems.

Finite set  $\mathcal{C}$  of anyonic charges:  $a, b, c \dots$

Unique “vacuum” charge, denoted  $I$  has trivial fusion and braiding with all particles.

Fusion rules: 
$$a \times b = \sum_c N_{ab}^c c$$

Fusion multiplicities  $N_{ab}^c$  are integers specifying the dimension of the fusion and splitting spaces  $V_{ab}^c, V_c^{ab}$

Hilbert space construct from state vectors associated with fusion/splitting channels of anyons.

Expressed diagrammatically:

$$\begin{array}{c} c \\ \uparrow \\ \swarrow \quad \searrow \\ a \quad b \end{array} = \langle a, b; c | \in V_{ab}^c$$

$$\begin{array}{c} a \quad b \\ \swarrow \quad \searrow \\ \uparrow \\ c \end{array} = |a, b; c\rangle \in V_c^{ab}$$

Inner product:

$$\begin{array}{c} c \\ \uparrow \\ \circlearrowleft \\ \swarrow \quad \searrow \\ a \quad b \\ \uparrow \\ c' \end{array} = \delta_{cc'} \begin{array}{c} c \\ \uparrow \end{array}$$

Associativity of fusing/splitting more than two anyons is specified by the unitary F-moves:

$$\begin{array}{c} a \\ \nearrow \\ \text{---} \\ \searrow \\ e \end{array}
 \begin{array}{c} b \\ \nearrow \\ \text{---} \\ \searrow \\ f \end{array}
 \begin{array}{c} c \\ \nearrow \\ \text{---} \\ \searrow \\ d \end{array}
 = \sum_f \left[ F_d^{abc} \right]_{ef}
 \begin{array}{c} a \\ \nearrow \\ \text{---} \\ \searrow \\ d \end{array}
 \begin{array}{c} b \\ \nearrow \\ \text{---} \\ \searrow \\ f \end{array}
 \begin{array}{c} c \\ \nearrow \\ \text{---} \\ \searrow \\ \end{array}$$

# Braiding

$$R^{ab} = \begin{array}{c} \nearrow \quad \nwarrow \\ a \quad \quad b \end{array} = \sum_c R_c^{ab} \begin{array}{c} \nwarrow \quad \nearrow \\ b \quad \quad a \\ \uparrow \\ c \\ \downarrow \\ a \quad \quad b \end{array}$$

Can be non-Abelian if there are multiple fusion channels  $c$

$$|\Psi_\alpha\rangle \mapsto U_{\alpha\beta}[R]|\Psi_\beta\rangle$$

Ising anyons or  $SU(2)_2$

-  $\nu = \frac{5}{2}$  FQH (Moore-Read '91)

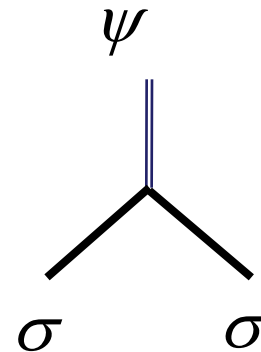
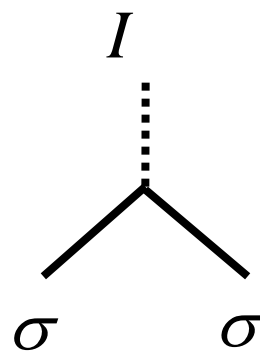
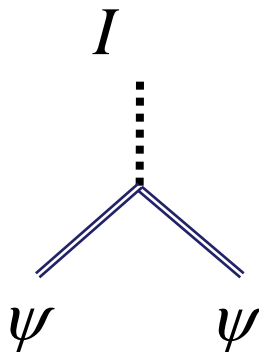
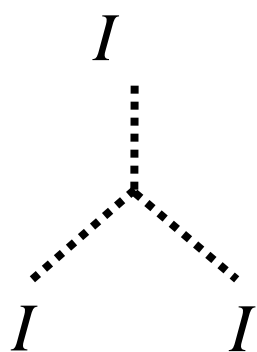
-  $\nu = \frac{12}{5}$  and other 2LL FQH? (PB and Slingerland '07)

- Kitaev honeycomb, topological insulators, ruthenates?

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Particle types:  $I$ ,  $\sigma$ ,  $\psi$  (a.k.a.  $0$ ,  $\frac{1}{2}$ ,  $1$ )

Fusion rules :



Fibonacci anyons or  $SO(3)_3$

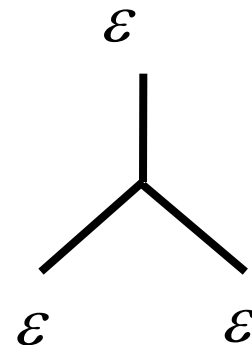
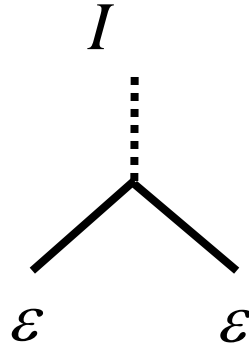
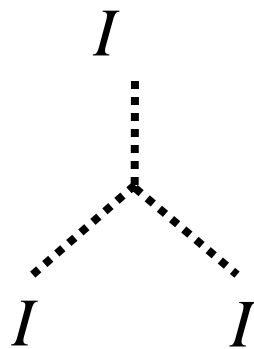
-  $\nu = \frac{12}{5}$  FQH? (Read - Rezayi '98)

- string nets? (Levin - Wen '04, Fendley et. al. '08)

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Particle types:  $I$ ,  $\varepsilon$  (a.k.a. 0, 1)

Fusion rules :





# Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)



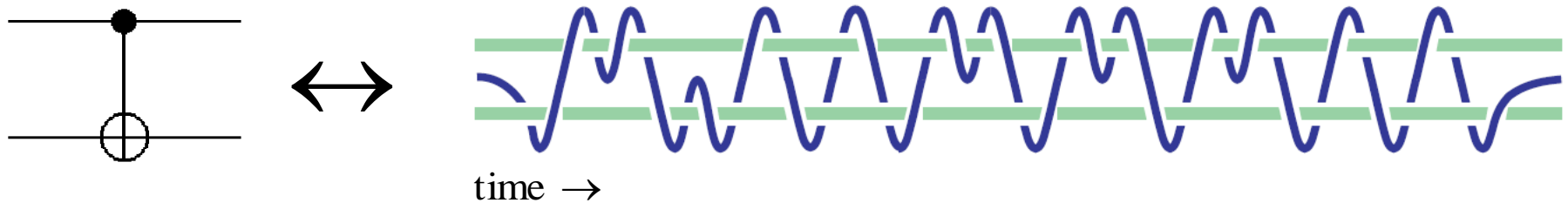
Topological Protection!

Ising:  $a = \sigma, c_0 = I, c_1 = \psi$

Fib:  $a = \varepsilon, c_0 = I, c_1 = \varepsilon$

# Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)



(Bonesteel, et. al.)

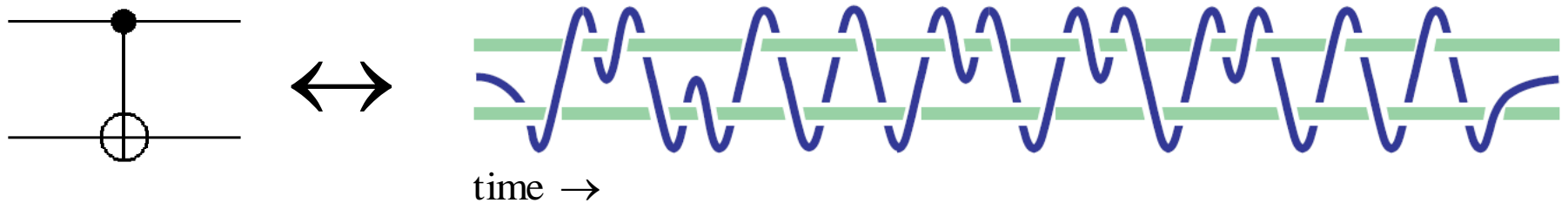
Is braiding computationally universal?

Ising: not quite  
(must be supplemented)

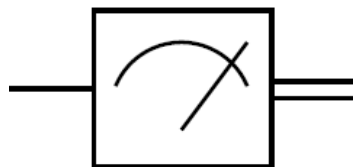
Fib: yes!

# Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)



(Bonesteel, et. al.)



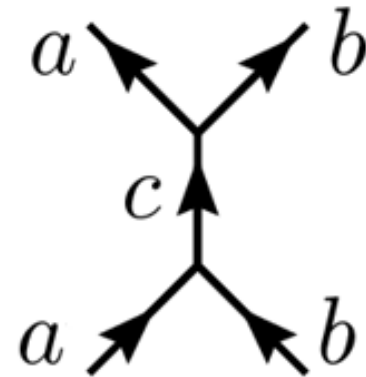
Topological Charge Measurement

# Topological Charge Measurement

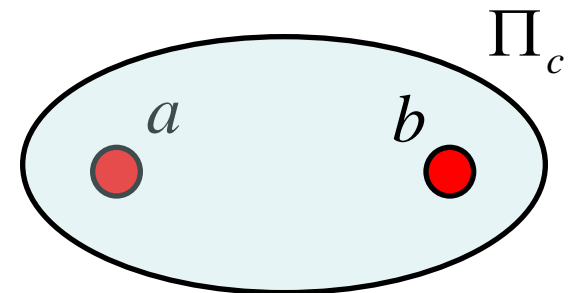
Projective (von Neumann)

e.g. loop operator measurements in lattice models, energy splitting measurement

$$\Pi_c = |a, b; c\rangle\langle a, b; c| =$$

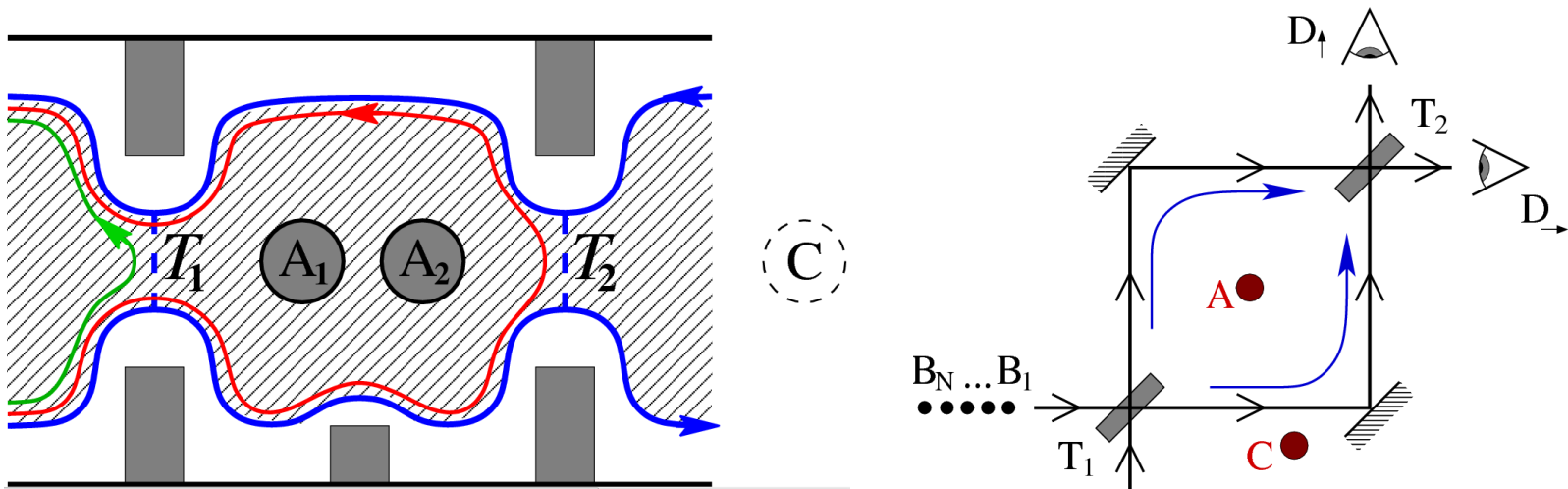


$$|\Psi\rangle \mapsto \frac{\Pi_c |\Psi\rangle}{\langle \Psi | \Pi_c | \Psi \rangle}$$



# Topological Charge Measurement

Interferometric (PB, Shtengel, Slingerland '07)  
e.g. 2PC FQH, and Anyonic Mach-Zehnder (idealized, not FQH)



Asymptotically characterized as projection of the target's anyonic charge AND decoherence of anyonic charge entanglement between the interior and exterior of the target region. (more later; ignore for now)

# Anyonic State Teleportation

(for projective measurement)

Entanglement Resource: maximally entangled anyon pair

$$|\bar{a}, a; I\rangle = \begin{array}{c} \bar{a} \quad a \\ \text{---} \cup \text{---} \end{array}$$

Want to teleport:  $|\psi\rangle = \begin{array}{c} a \\ | \\ \text{---} \psi \text{---} \end{array}$

Form:  $|\psi\rangle_1 |\bar{a}, a; I\rangle_{23} = \begin{array}{c} a \quad \bar{a} \quad a \\ | \quad \text{---} \cup \text{---} \\ \text{---} \psi \text{---} \end{array}$

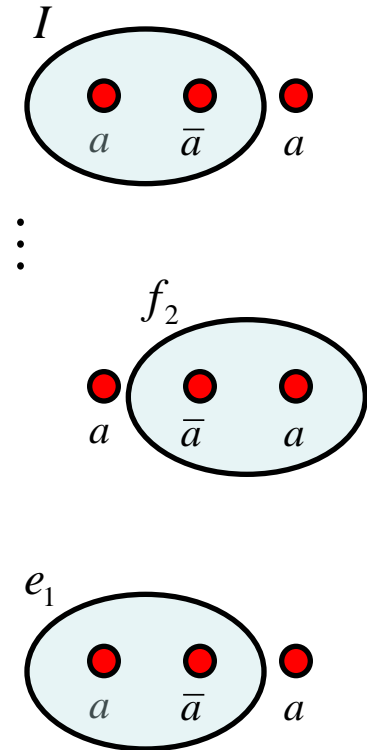
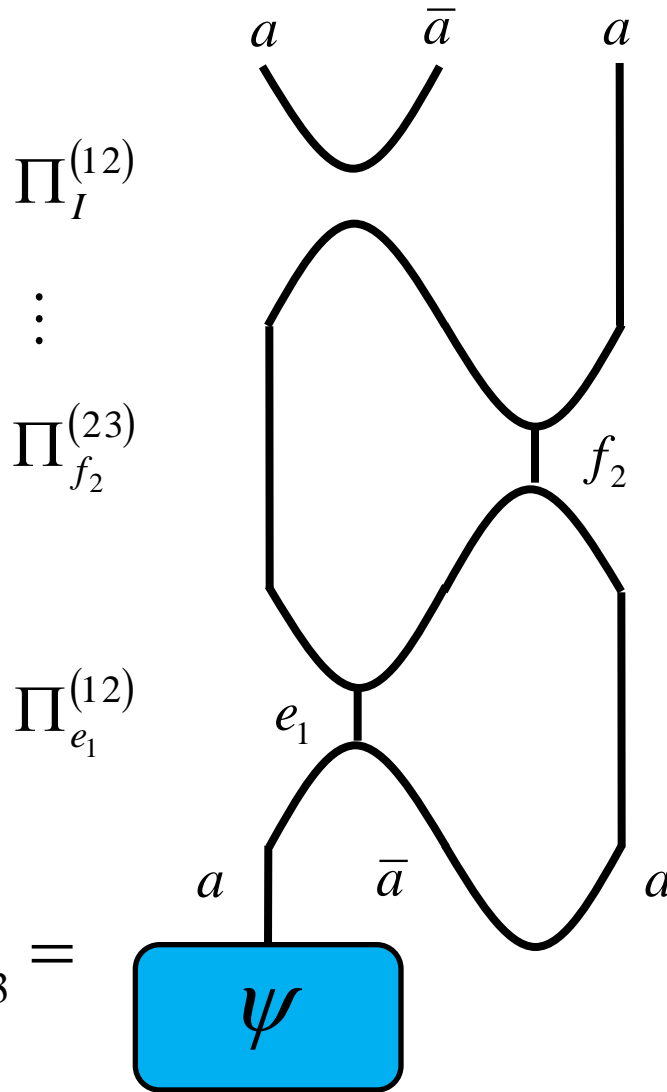
and perform **Forced Measurement** on anyons 12

# Anyonic State Teleportation

Forced  
Measurement  
(projective)

$\check{\Pi}_I^{(12)} :$

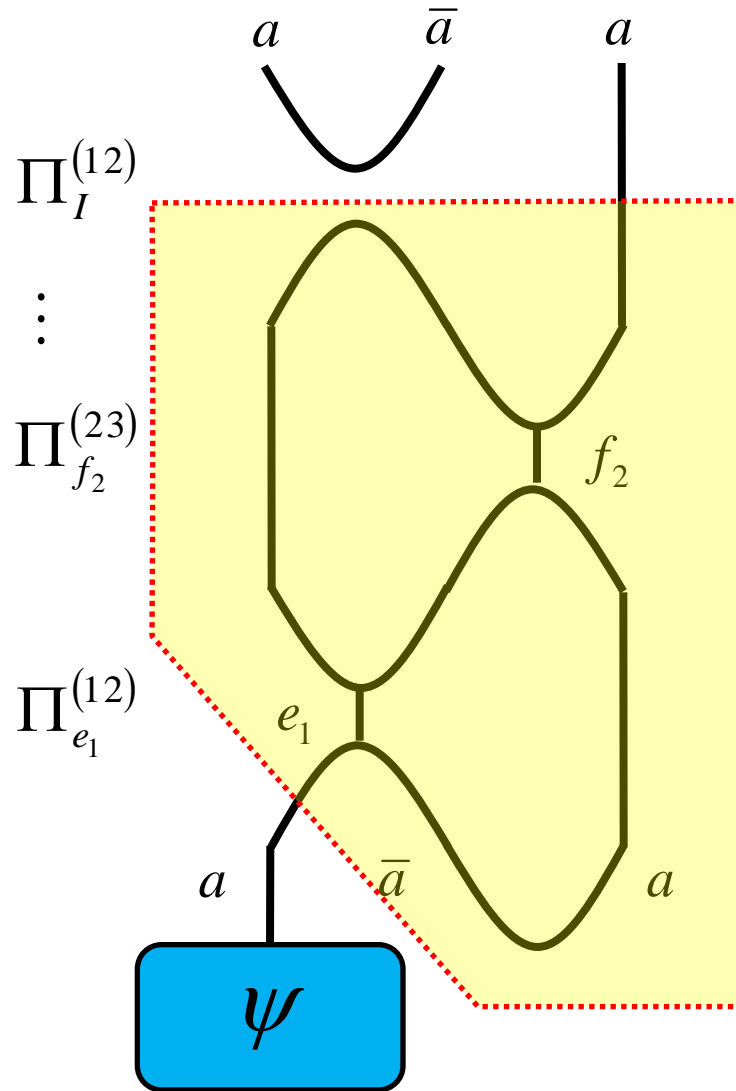
$$|\psi\rangle_1 |\bar{a}, a; I\rangle_{23} =$$



# Anyonic State Teleportation

Forced  
Measurement  
(projective)

$\check{\Pi}_I^{(12)} :$





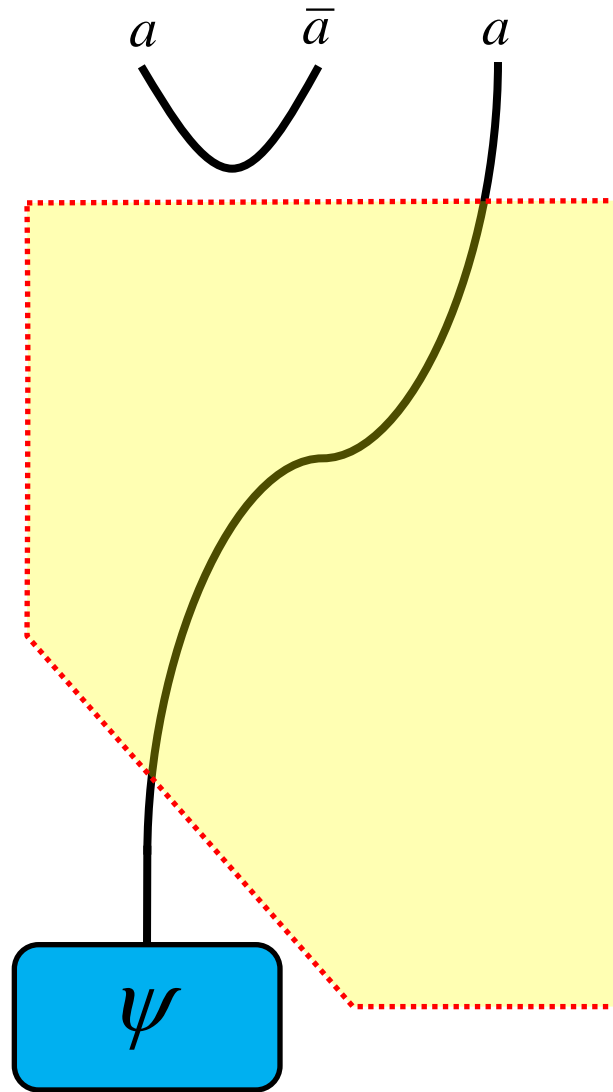
# Anyonic State Teleportation

Forced  
Measurement  
(projective)

$$\check{\Pi}_I^{(12)} \cong \Pi_I^{(12)} :$$

$$|\psi\rangle_1 |\bar{a}, a; I\rangle_{23}$$

$$\mapsto |a, \bar{a}; I\rangle_{12} |\psi\rangle_3 =$$

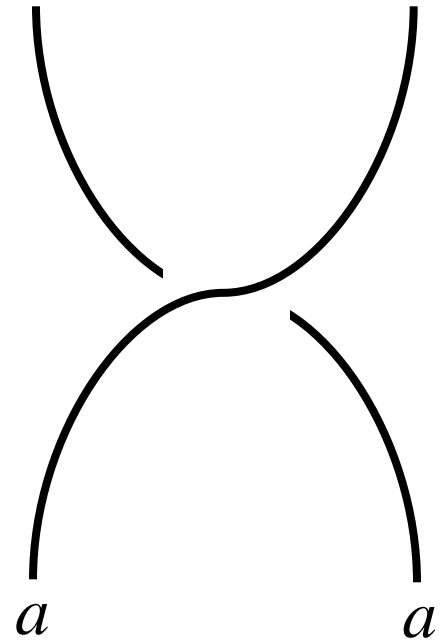


“Success” occurs with probability  $\geq \frac{1}{d_a^2}$  for each repeat try.

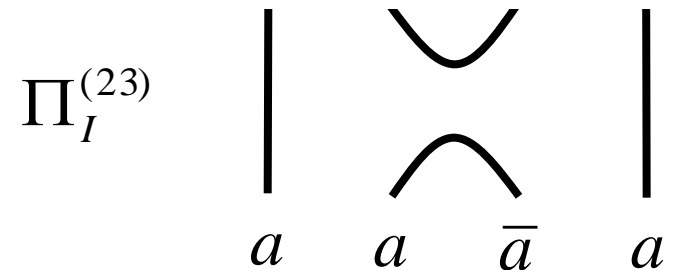
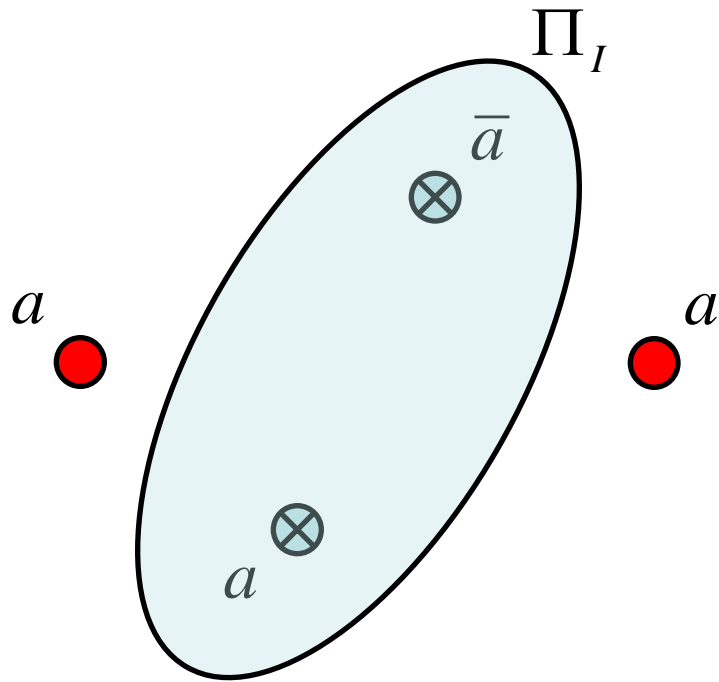
What good is this if we want to braid computational anyons?



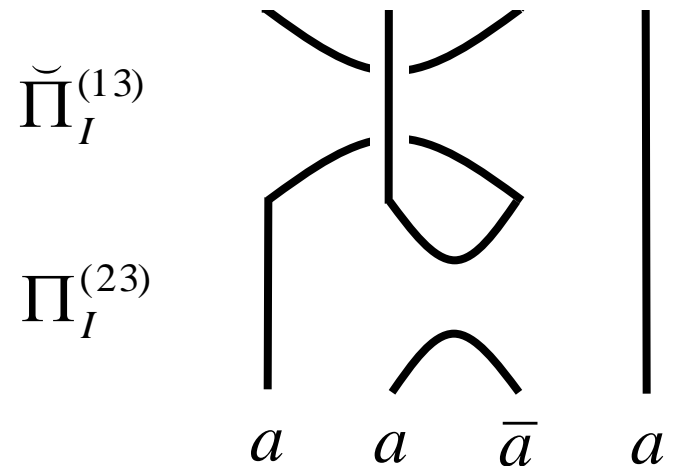
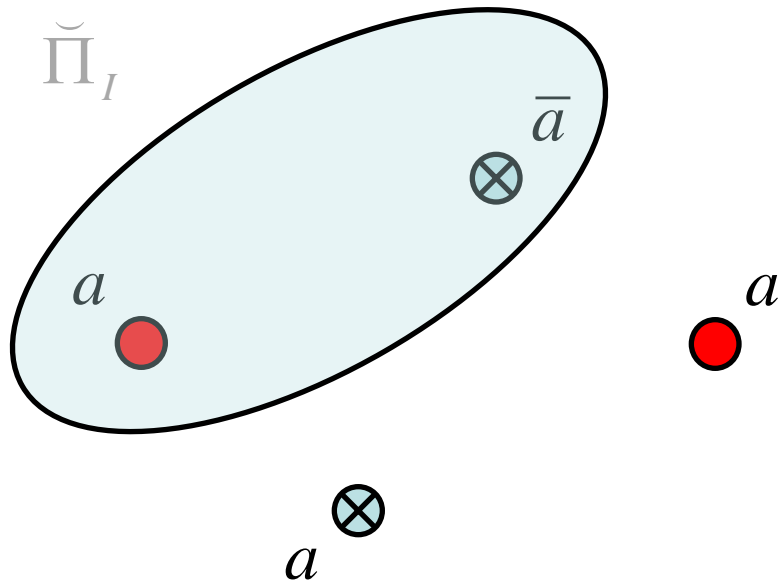
$R =$



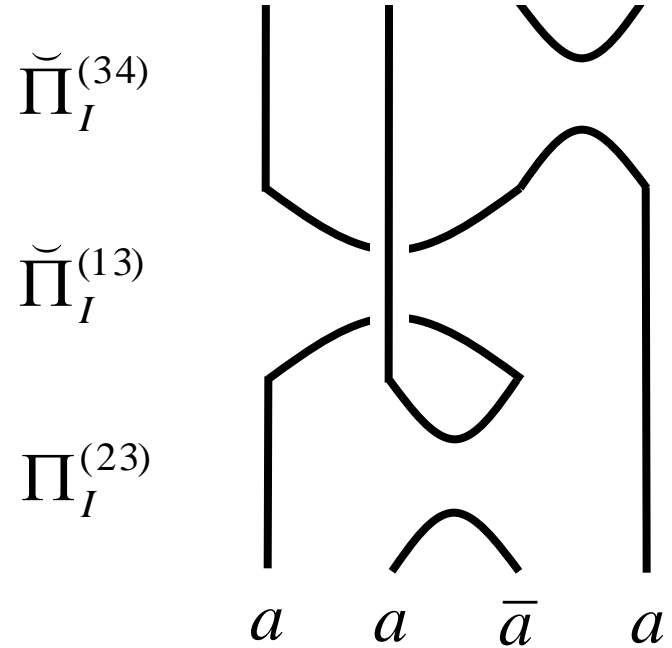
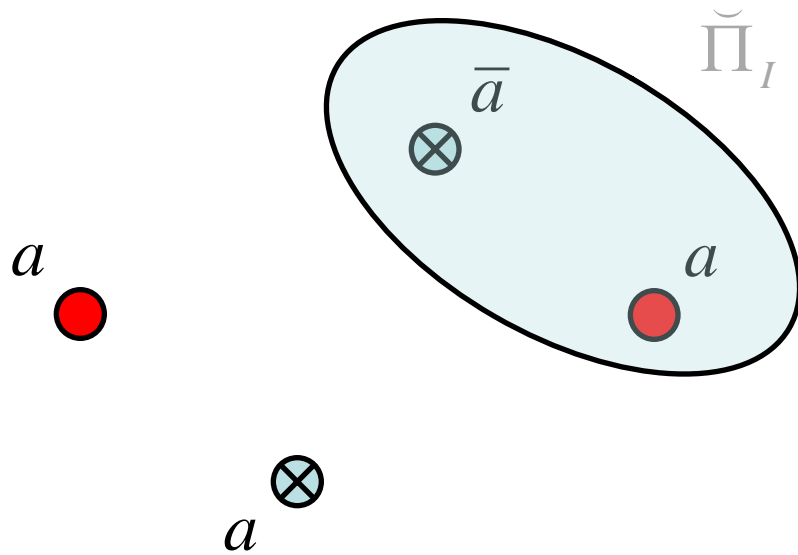
Use a maximally entangled pair and “forced measurements” for a series of teleportations



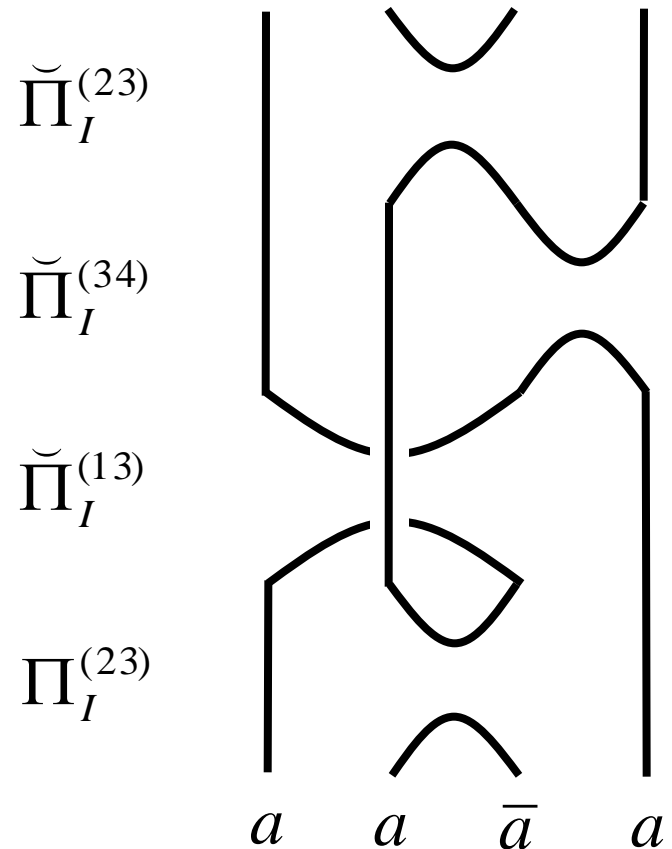
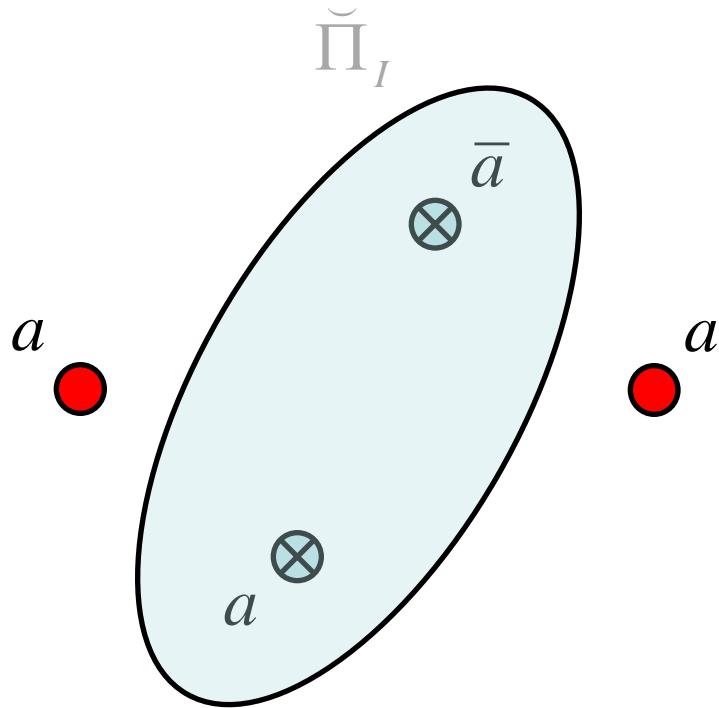
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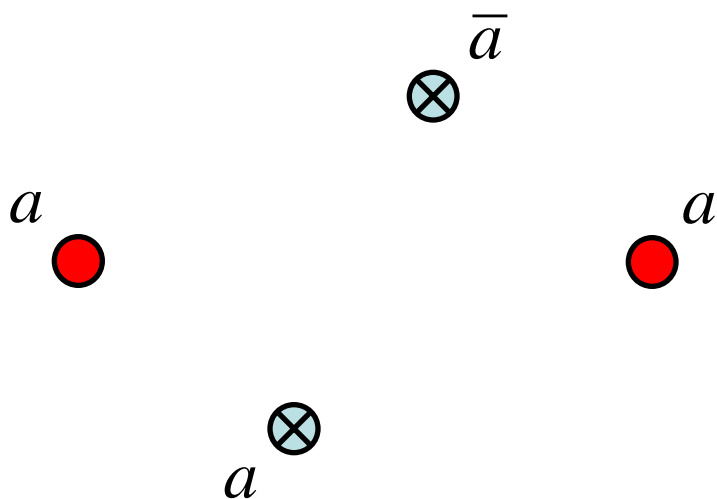
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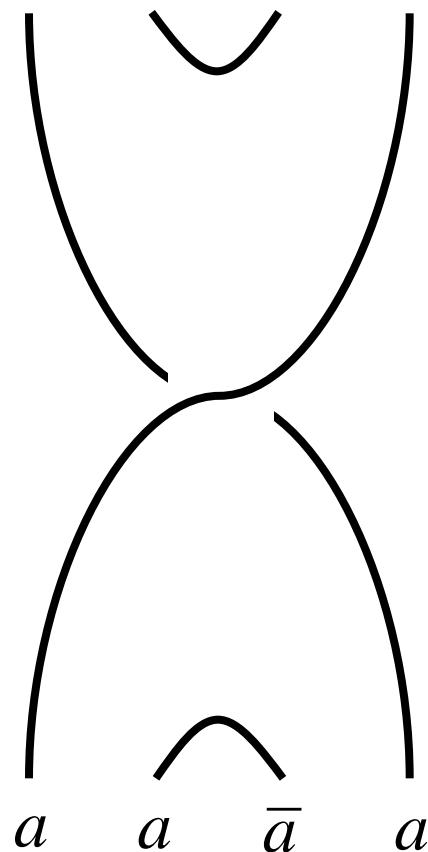
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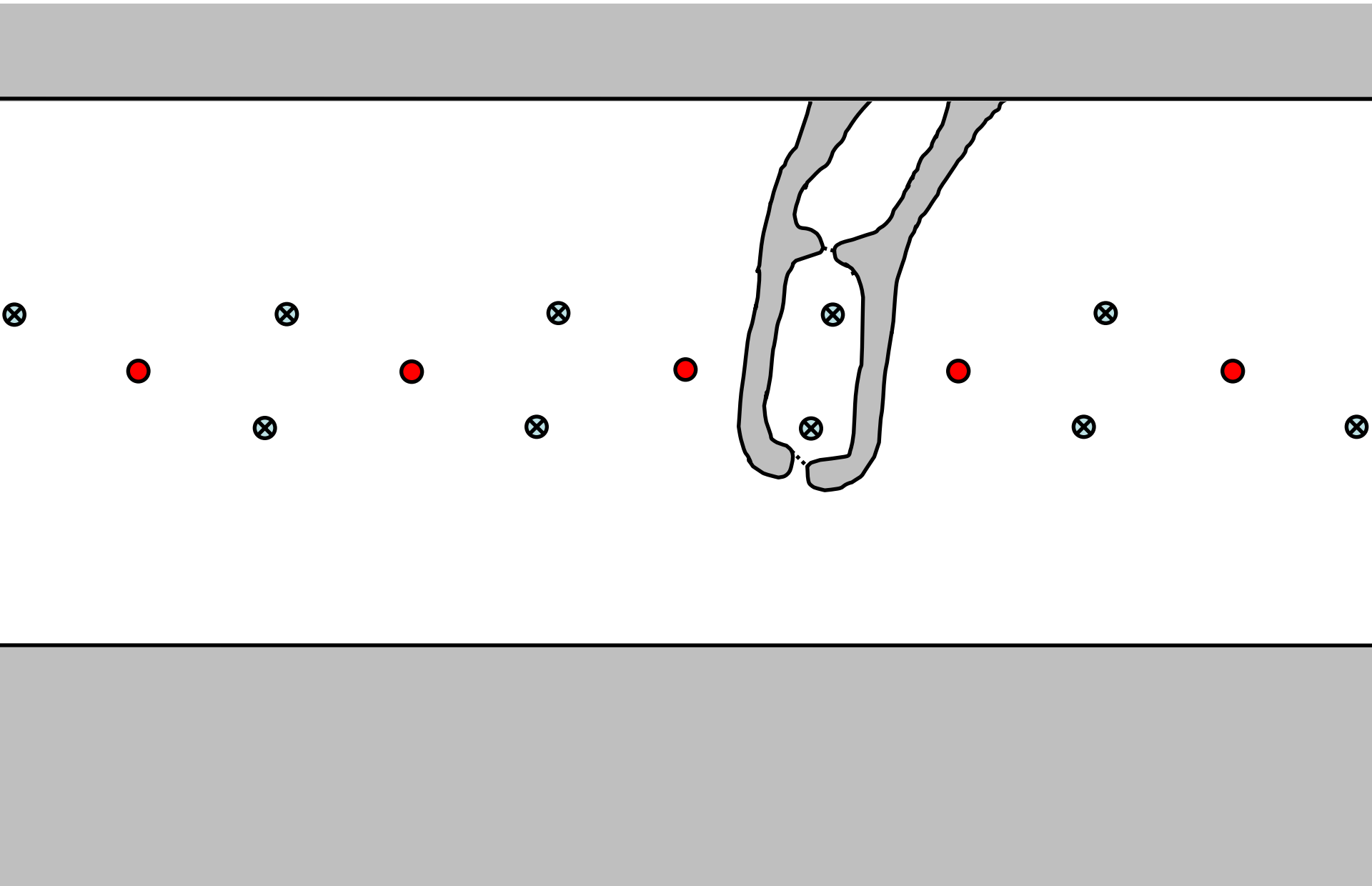
# Measurement Simulated Braiding!



$$R^{(14)} \cong \check{\Pi}_I^{(23)} \check{\Pi}_I^{(34)} \check{\Pi}_I^{(13)} \Pi_I^{(23)} =$$

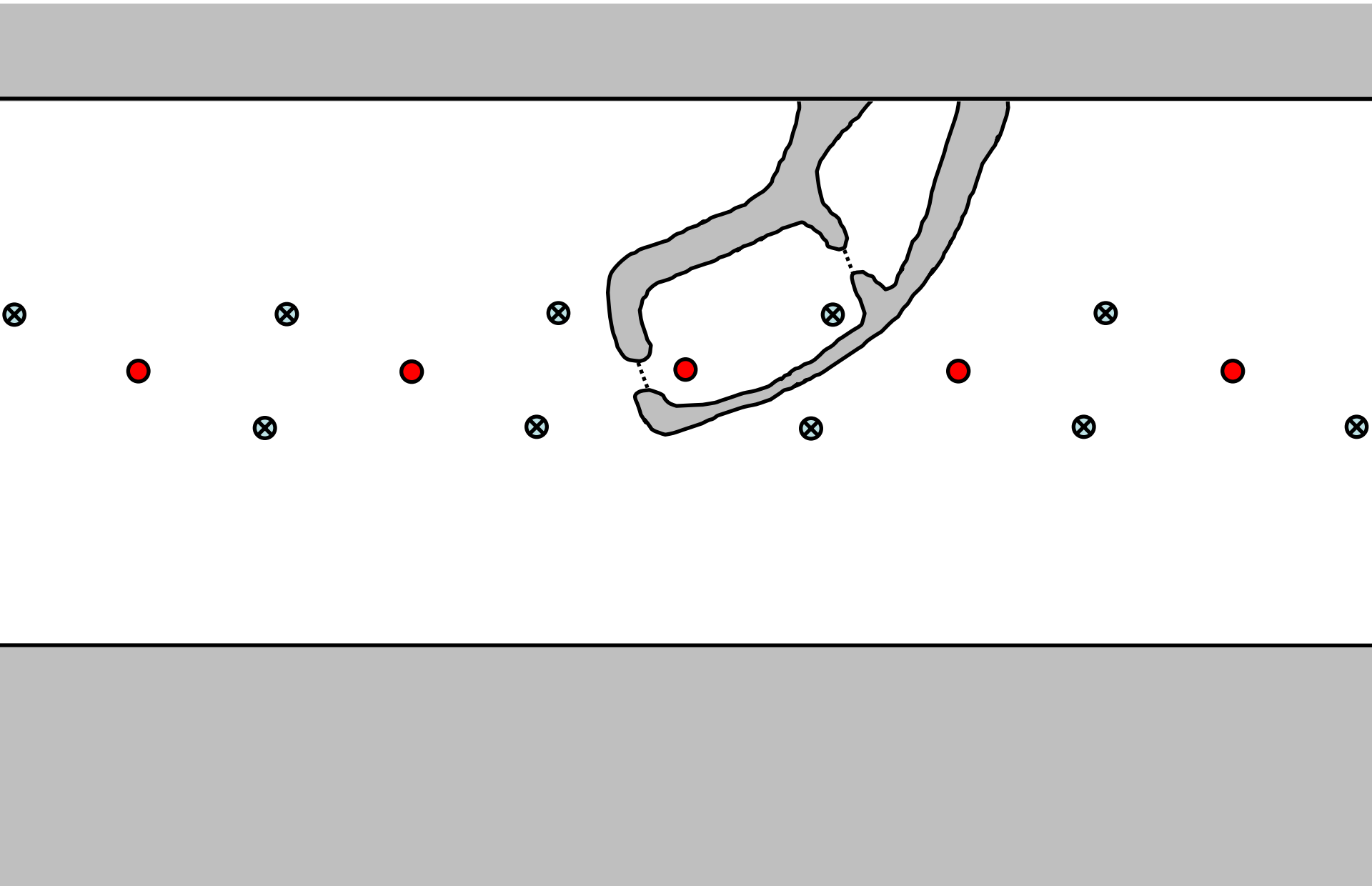


in FQH, for example

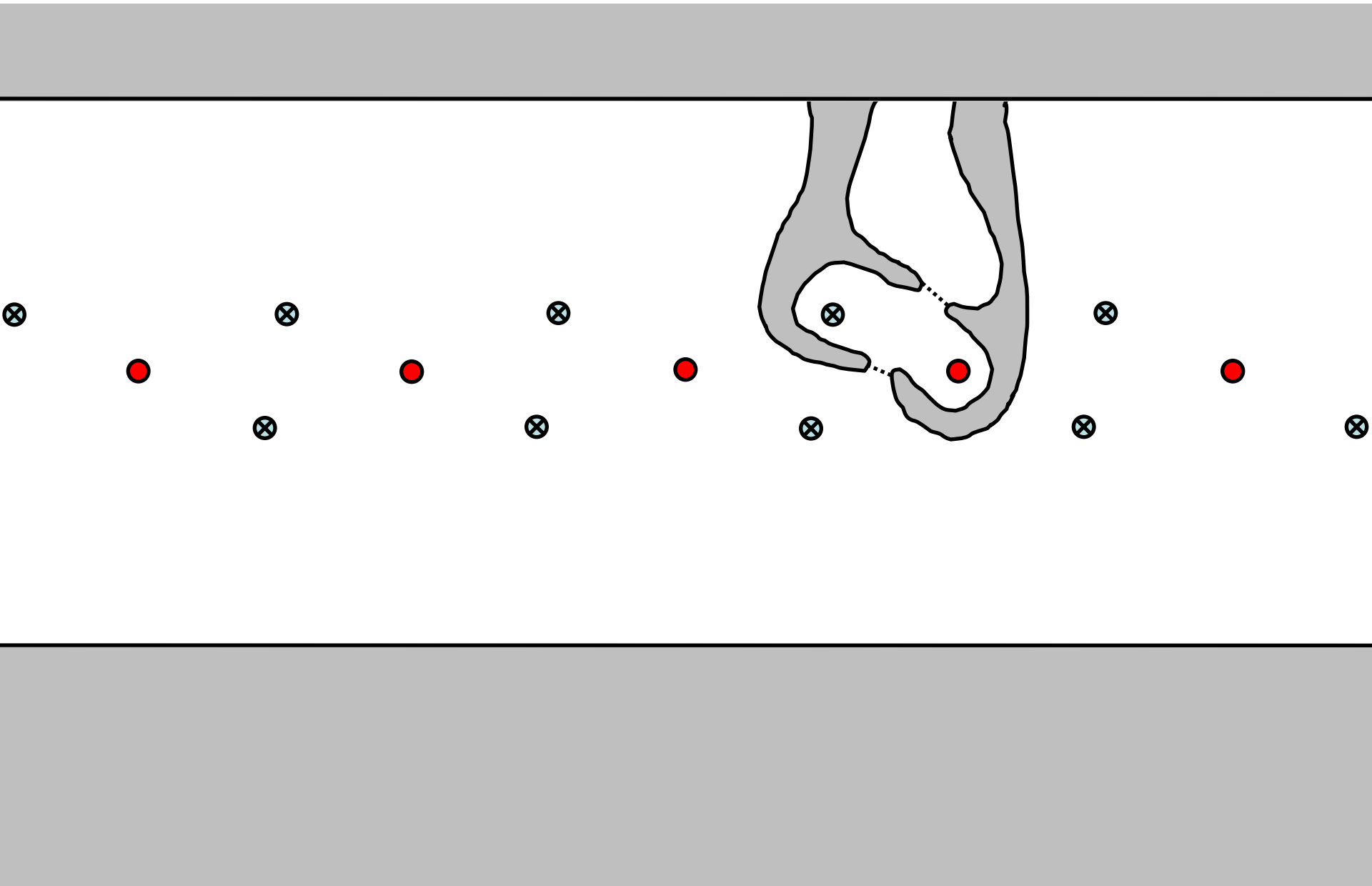




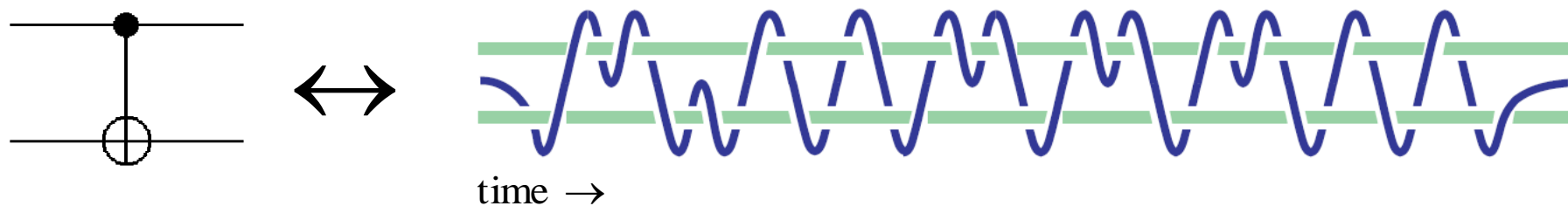
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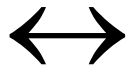
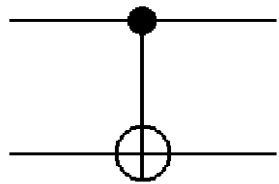
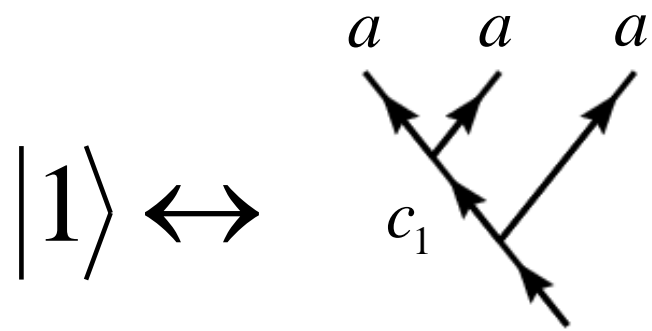
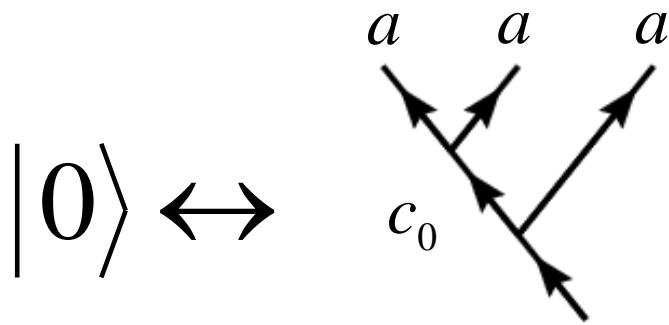
# Topological Quantum Computation



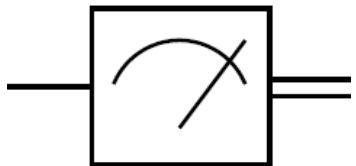
 measurement simulated braiding



# Measurement-Only Topological Quantum Computation



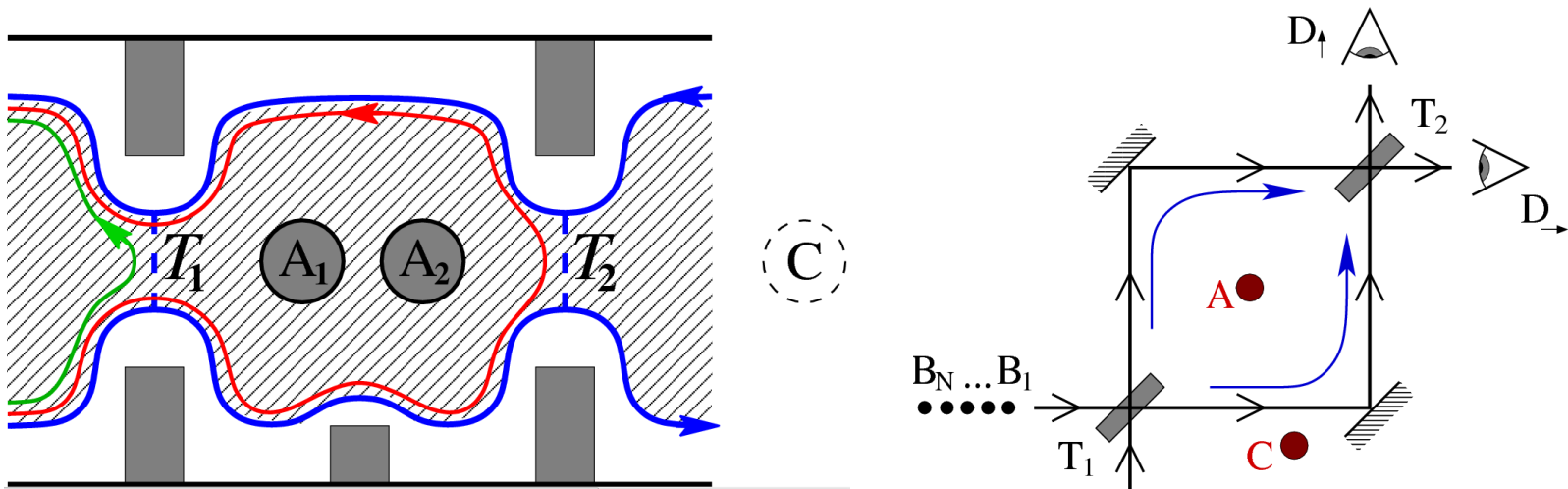
Topological Charge Measurement



Topological Charge Measurement

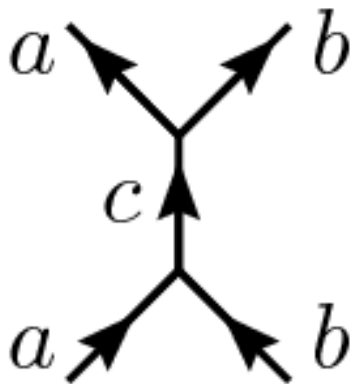
# Topological Charge Measurement

Interferometric (PB, Shtengel, Slingerland '07)  
e.g. 2PC FQH, and Anyonic Mach-Zehnder (idealized, not FQH)

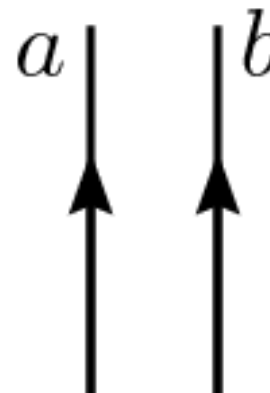


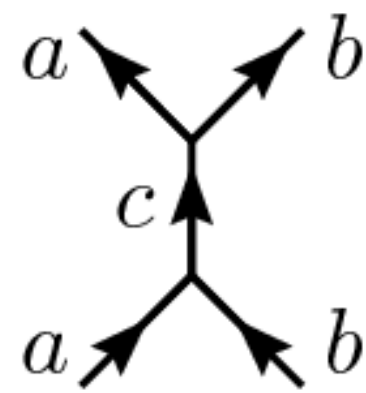
Asymptotically characterized as projection of the target's anyonic charge AND decoherence of anyonic charge entanglement between the interior and exterior of the target region.

# Interferometrical Decoherence of Anyonic Charge Entanglement

$$\rho = |a, b; c\rangle\langle a, b; c| =$$


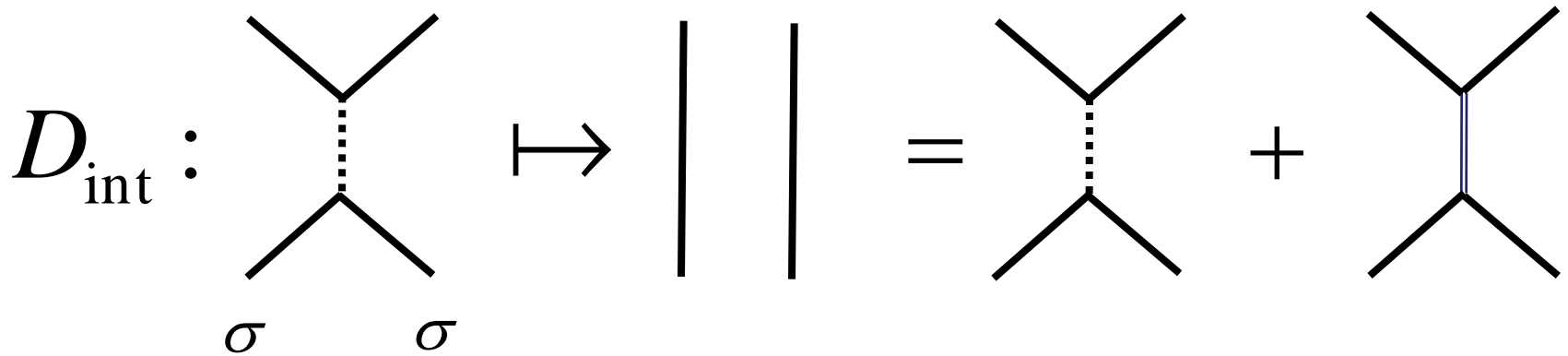
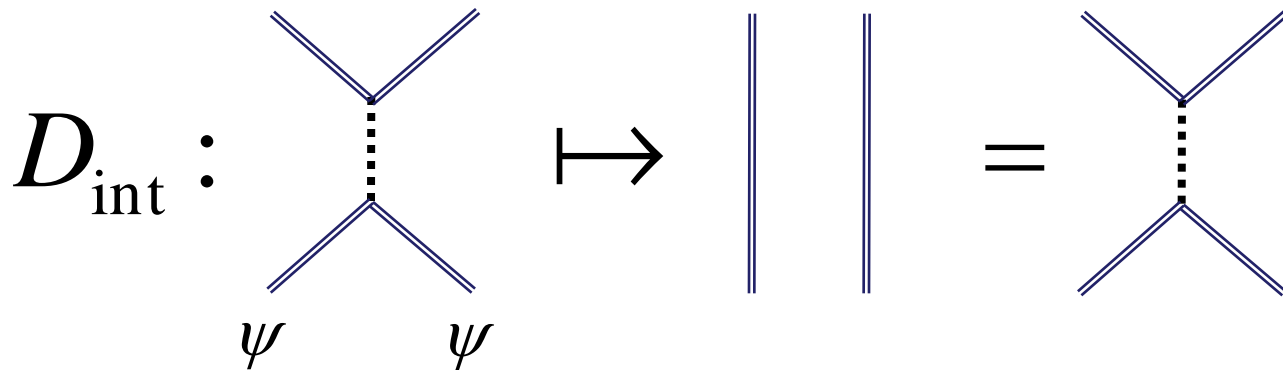
For  $a$  inside the interferometer and  $b$  outside:

$$D_{\text{int}} : \rho \mapsto$$


$$= \sum_c$$


# Interferometrical Decoherence

Ising:



# Interferometrical Decoherence

Fibonacci:

$$D_{\text{int}} : \begin{array}{c} \diagup \quad \diagdown \\ \text{---} \\ \text{---} \\ \diagdown \quad \diagup \\ \varepsilon \quad \varepsilon \end{array} \quad \mapsto \quad \begin{array}{c} | \\ | \end{array} \quad = \quad \begin{array}{c} \diagup \quad \diagdown \\ \text{---} \\ \diagdown \quad \diagup \end{array} \quad + \quad \begin{array}{c} \diagup \quad \diagdown \\ | \\ \diagdown \quad \diagup \end{array}$$



# Measurement Generated Braiding!

Using Interferometric Measurements is similar but more complicated, requiring the density matrix description.

The resulting “forced measurement” procedure must include an additional measurement (of 8 or fewer anyons, i.e. still bounded size) in each teleportation attempt to ensure the overall charge of the topological qubits being acted upon remains trivial.

Note: For the Ising model TQC qubits, interferometric measurements are projective.

# Ising

vs

# Fibonacci

(in FQH)

- Braiding not universal  
(needs one gate supplement)



$\Delta_{\nu=5/2} \sim 600$  mK



Braids = Natural gates  
(braiding = Clifford group)



No leakage from braiding



Projective MOTQC  
(2 anyon measurements)

- Measurement difficulty  
distinguishing  $I$  and  $\psi$   
(precise phase calibration)



Braiding is universal

- $\Delta_{\nu=12/5} \sim 70$  mK
- Braids = Unnatural gates  
(see Bonesteel, et. al.)
- Inherent leakage errors  
(from entangling gates)
- Interferometrical MOTQC  
(2,4,8 anyon measurements)



Robust measurement  
distinguishing  $I$  and  $\varepsilon$   
(amplitude of interference)

# Conclusion

- Quantum state teleportation and entanglement resources have anyonic counterparts.
- Bounded, adaptive, non-demolitional measurements can generate the braiding transformations used in TQC.
- Stationary computational anyons hopefully makes life easier for experimental realization.
- Experimental realization of FQH double point-contact interferometers is at hand.