# Exercises CFT-course fall 2008, set 4.

Due on friday, october 31, 2008.

1. The OPE of descendent fields.

   Show that the OPE of $T(z)$ with $\phi^{-1}(w) = \partial\phi(w)$, where $\phi(w)$ is a primary field of dimension $h$, takes the form

   $$T(z)\partial\phi(w) = \frac{2h\phi(w)}{(z-w)^3} + \frac{(h+1)\partial\phi(w)}{(z-w)^2} + \frac{\partial^2\phi(w)}{z-w} + \text{reg} \; .$$

2. Highest weight condition.

   Show that the highest weight condition, namely $L_n|\chi\rangle = 0$ for all $n > 0$ is implied by the conditions $L_1|\chi\rangle = L_2|\chi\rangle = 0$.

3. Null vector at level 3.

   Find the explicit expression for the null vector at level 3. Answer:

   $$|\chi_{1,3}\rangle = \big(L_{-3} - \frac{2}{h}L_{-2}L_{-1} + \frac{1}{h(h+1)}(L_{-1})^3\big)|h\rangle$$

   Determine the corresponding central charge $c$ as a function of $h$.

4. The Kac determinant.

   Calculate the Kac determinant at level 2, i.e the determinant of the Gramm-matrix of the states $(L_{-1})^2|h\rangle$ and $L_{-2}|h\rangle$.

5. A little on partitions.

   Let $p_d(n)$ be the number of partitions of $n \geq 0$ into *distinct* parts. Thus, f.i. $p_d(5) = 3$, namely $5 = 1+4$, $5 = 2+3$, $5 = 5$.

   Let $p_o(n)$ be the number of partitions of $n \geq 0$ into *odd* parts. Thus, f.i. $p_o(5) = 3$, namely $5 = 1+1+1+1+1$, $5 = 1+1+3$, $5 = 5$.

   Show that $p_d(n) = p_o(n)$ by obtaining their generating functions, and showing that they are equal.

   Hint: $\prod_{n=1}^{\infty}(1+q^n)(1-q^{2n-1}) = 1$, for $|q| < 1$.