Exercises CFT-course fall 2008, set 1.

Due on wednesday, october 8, 2008.

1. Renormalization group flow in one dimension.

a. The one-dimensional Ising model in a (reduced) magnetic field is described by the hamiltonian

$$H = -K \sum_j s_j s_{j+1} - h \sum_j s_j \ ,$$

where $s_j = \pm 1$. Perform a renormalization group transformation (with scale factor $b = 2$) by summing over every other spin. Discuss the renormalization group flows in the (x, y)-plane, where $x = e^{-2K}$ and $y = e^h$.

b. Let t_j be a 'spin' at site j which can take the values $t_j = 1, 2, 3$. The one-dimensional three states Potts model is described by the hamiltonian

$$H = -K \sum_j \delta_{t_j, t_{j+1}}$$

Perform a similar renormalization group transformation as in a., and derive the flow equation. Show that there are no non-trivial fixed points.

2. The infinitesimal form of the special conformal transformation.

Write the most general from of the quadratic part of ε_μ, namely $\varepsilon_\mu = c_{\mu\nu\rho} x^\nu x^\rho$, to derive the infinitesimal transformation

$$x'^\mu = x^\mu + b^\mu x^2 - 2x^\mu (b \cdot x) \ ,$$

and give the explicit form of b^μ.

3. The (finite) special conformal transformations (in d dimensions) have the form

$$\mathbf{x}' = \frac{\mathbf{x} + \mathbf{b}x^2}{1 + 2\mathbf{b} \cdot \mathbf{x} + b^2 x^2} \ .$$

a. Derive the scale factor $\Omega(\mathbf{x})$ for this transformation: $\Omega(\mathbf{x}) = (1 + 2\mathbf{b} \cdot \mathbf{x} + b^2 x^2)^2$

b. Show that $|\mathbf{x}'_1 - \mathbf{x}'_2|^2 = \frac{|\mathbf{x}_1 - \mathbf{x}_2|^2}{\gamma_1 \gamma_2}$, where $\gamma_i = (1 + 2\mathbf{b} \cdot \mathbf{x}_i + b^2 x_i^2)$

c. Show that the special conformal transformations leave angles invariant.

4. Show that the three point functions of quasi-primary fields have the following form

$$\langle \phi_1(x_1) \phi_2(x_2) \phi_3(x_3) \rangle = \frac{C_{123}}{x_{12}^{h_1+h_2-h_3} x_{23}^{h_2+h_3-h_1} x_{13}^{h_1+h_3-h_2}}$$

The h_i are the scaling dimensions of the fields ϕ_i.

5. Show that the n point functions of quasi-primary fields satisfy

$$\sum_i \partial_{z_i} \langle \phi_1(z_1) \cdots \phi_n(z_n) \rangle = 0$$
$$\sum_i (z_i \partial_{z_i} + h_i) \langle \phi_1(z_1) \cdots \phi_n(z_n) \rangle = 0$$
$$\sum_i (z_i^2 \partial_{z_i} + 2h_i z_i) \langle \phi_1(z_1) \cdots \phi_n(z_n) \rangle = 0 \ ,$$

by considering the infinitesimal form of the global conformal transformations.

6. (Optional). On the course website, http://www.nordita.org/˜ardonne/cft-course.html, there are three pictures. What do they correspond to?