

Exercises CFT-course fall 2023, set 4.

1. The two point function on the cylinder.
 - a. Calculate the two point function of a primary operator $\phi(z, \bar{z})$ with scaling dimensions (h, \bar{h}) on the cylinder, by using the conformal map $w = \frac{L \log z}{2\pi}$. Result:

$$\langle \phi(w_1, \bar{w}_1) \phi(w_2, \bar{w}_2) \rangle = \left(\frac{2\pi}{L} \right)^{2h+2\bar{h}} \left(2 \sinh(\pi(w_1 - w_2)/L) \right)^{-2h} \left(2 \sinh(\pi(\bar{w}_1 - \bar{w}_2)/L) \right)^{-2\bar{h}}$$

- b. Let $w = t + iu$. Analyze the result of a. in the limits $t_1 - t_2 \ll L$ and $t_1 - t_2 \gg L$ and discuss the result. For simplicity, you may assume that $h = \bar{h}$.
2. The massless free boson ϕ has the action

$$S = \frac{g}{2} \int dx^2 \partial_\mu \phi \partial^\mu \phi .$$

- a. Determine the propagator of the free massless boson theory:

$$\langle \phi(z, \bar{z}) \phi(w, \bar{w}) \rangle = \frac{-1}{4\pi g} (\ln(z - w) + \ln(\bar{z} - \bar{w})) + \text{const.}$$

- b. Determine the form of the stress-energy tensor, namely $T(z) = -2\pi g : \partial\phi(z)\partial\phi(z) :$
 - c. Calculate the OPE $T(z)T(w)$.
 - d. Show that the vertex operator $V =: e^{i\alpha\phi} : (z)$ is a primary field of dimension $h_\alpha = \frac{\alpha^2}{8\pi g}$.

3. The massless free fermion.

The action of the massless free fermion can be written as

$$S = g \int dx^2 (\bar{\psi} \partial \bar{\psi} + \psi \bar{\partial} \psi) ,$$

where ψ and $\bar{\psi}$ are the two components of the spinor Ψ . It is given that the equations of motion are $\partial \bar{\psi} = \bar{\partial} \psi = 0$, while the propagator takes the form $\langle \psi(z, \bar{z}) \psi(w, \bar{w}) \rangle = \frac{1}{2\pi g} \frac{1}{z-w}$. The stress-energy tensor can be written as $T(z) = -\pi g : \psi(z) \partial \psi(z) :$

- a. Show that $\psi(z)$ is a primary field.
 - b. Calculate the OPE $T(z)T(w)$.