

Exercises CFT-course fall 2023, set 2.

1. The infinitesimal form of the special conformal transformations.

Write the most general form of the quadratic part of ε_μ , namely $\varepsilon_\mu = c_{\mu\nu\rho}x^\nu x^\rho$, to derive the infinitesimal transformation

$$x'^\mu = x^\mu + b^\mu x^2 - 2x^\mu (b \cdot x) ,$$

and give the explicit form of b^μ in terms of the tensor c .

2. The (finite) special conformal transformations (in d dimensions) have the form

$$\mathbf{x}' = \frac{\mathbf{x} + \mathbf{b}x^2}{1 + 2\mathbf{b} \cdot \mathbf{x} + b^2 x^2} .$$

- a. Derive the scale factor $\Omega(\mathbf{x})$ for this transformation: $\Omega(\mathbf{x}) = (1 + 2\mathbf{b} \cdot \mathbf{x} + b^2 x^2)^2$
- b. Show that $|\mathbf{x}'_1 - \mathbf{x}'_2|^2 = \frac{|\mathbf{x}_1 - \mathbf{x}_2|^2}{\gamma_1 \gamma_2}$, where $\gamma_i = (1 + 2\mathbf{b} \cdot \mathbf{x}_i + b^2 x_i^2)$
3. ‘Quasi-primary’ fields transform under (passive) global conformal transformations in d dimensions as follows

$$\phi_i(\mathbf{x}) \rightarrow \left| \frac{\partial \mathbf{x}'}{\partial \mathbf{x}} \right|^{\frac{h_i}{d}} \phi_i(\mathbf{x}') ,$$

where the h_i are the scaling dimensions of the fields ϕ_i .

Show that the three point functions of quasi-primary fields have the following form

$$\langle \phi_1(\mathbf{x}_1) \phi_2(\mathbf{x}_2) \phi_3(\mathbf{x}_3) \rangle = \frac{C_{123}}{x_{12}^{h_1+h_2-h_3} x_{23}^{h_2+h_3-h_1} x_{13}^{h_1+h_3-h_2}} ,$$

where we introduced the notation $x_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$.