

Exercises CFT-course fall 2018, set 3.

Due on friday, november 23, 2018.

1. The conformal ward identity.

a. Carefully derive the conformal ward identity

$$\delta \langle \Phi_1(z_1) \cdots \Phi_n(z_n) \rangle = \frac{1}{2\pi i} \oint_C dz \varepsilon(z) \langle T(z) \Phi_1(z_1) \cdots \Phi_n(z_n) \rangle + c.c. .$$

b. Show that for primary fields, this gives rise to the equation

$$\langle T(z) \Phi_1(z_1) \cdots \Phi_n(z_n) \rangle = \sum_i \left(\frac{h_i}{(z - z_i)^2} + \frac{\partial_{z_i}}{(z - z_i)} \right) \langle \Phi_1(z_1) \cdots \Phi_n(z_n) \rangle .$$

2. Show that the n point functions of quasi-primary fields satisfy

$$\begin{aligned} \sum_i \partial_{z_i} \langle \phi_1(z_1) \cdots \phi_n(z_n) \rangle &= 0 \\ \sum_i (z_i \partial_{z_i} + h_i) \langle \phi_1(z_1) \cdots \phi_n(z_n) \rangle &= 0 \\ \sum_i (z_i^2 \partial_{z_i} + 2h_i z_i) \langle \phi_1(z_1) \cdots \phi_n(z_n) \rangle &= 0 , \end{aligned}$$

by considering the infinitesimal form of the global conformal transformations.

3. The Schwartzian derivative.

a. Show that the Schwartzian derivative

$$\{w; z\} = \frac{w'''}{w'} - \frac{3}{2} \left(\frac{w''}{w'} \right)^2 ,$$

(with $w' = \frac{dw}{dz}$ etc), is zero if and only if $w(z) = \frac{az+b}{cz+d}$. Start by solving the first order differential equation for (w''/w') , and proceed by solving for $v = w'$.

b. Show that for successive transformations $z \rightarrow w \rightarrow u$, one has the following composition rule

$$\{u; z\} = \{w; z\} + \left(\frac{dw}{dz} \right)^2 \{u; w\}$$