Exercises CFT-course fall 2018, set 2.

1. The infinitesimal form of the special conformal transformations.

   Write the most general from of the quadratic part of ε_μ, namely $\varepsilon_\mu = c_{\mu\nu\rho}x^\nu x^\rho$, to derive the infinitesimal transformation

   $$x'^\mu = x^\mu + b^\mu x^2 - 2x^\mu(b \cdot x) ,$$

   and give the explicit form of b^μ in terms of the tensor c.

2. The (finite) special conformal transformations (in d dimensions) have the form

   $$\mathbf{x}' = \frac{\mathbf{x} + \mathbf{b}x^2}{1 + 2\mathbf{b}\cdot\mathbf{x} + b^2x^2} .$$

a. Derive the scale factor $\Omega(\mathbf{x})$ for this transformation: $\Omega(\mathbf{x}) = (1 + 2\mathbf{b}\cdot\mathbf{x} + b^2x^2)^2$

b. Show that $|\mathbf{x}'_1 - \mathbf{x}'_2|^2 = \frac{|\mathbf{x}_1 - \mathbf{x}_2|^2}{\gamma_1\gamma_2}$, where $\gamma_i = (1 + 2\mathbf{b}\cdot\mathbf{x}_i + b^2x_i^2)$

3. 'Quasi-primary' fields transform under (passive) global conformal transformations in d dimensions as follows

   $$\phi_i(\mathbf{x}) \to \left|\frac{\partial \mathbf{x}'}{\partial \mathbf{x}}\right|^{\frac{h_i}{d}} \phi_i(\mathbf{x}') ,$$

   where the h_i are the scaling dimensions of the fields ϕ_i.
   Show that the three point functions of quasi-primary fields have the following form

   $$\langle\phi_1(\mathbf{x}_1)\phi_2(\mathbf{x}_2)\phi_3(\mathbf{x}_3)\rangle = \frac{C_{123}}{x_{12}^{h_1+h_2-h_3}x_{23}^{h_2+h_3-h_1}x_{13}^{h_1+h_3-h_2}} ,$$

   where we introduced the notation $x_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$.