1. We study the zero-energy bound states of domain wall in the context of a one-dimensional, two-component Dirac fermion. That is, we consider the wave equation for the two-component spinor $\psi(x)$,

$$\left(-i\sigma^x \nabla_x - m(x)\sigma^z\right)\psi(x) = E\psi(x) \ , \tag{1}$$

   where at first, $m(x) = m_0 \tanh(\frac{x}{\xi\sqrt{2}})$.

   a. (3 p) Show that this model exhibits a zero-mode located at the origin, and find its shape explicitly.

   b. (2 p) We now assume an arbitrary 'mass' function $m(x)$. For which functions $m(x)$ is there a zero mode? Give an expression for their shape when they exist.

2. (5 p) Coleman exercise 11.3.

3. (3 p) Coleman exercise 11.4.