Quantum Field Theory for Condensed Matter - 2018
Exercise Set 2 (13 points)
Due date: monday, may 7th

1. (3 p) Do exercise 5.3 from Coleman.

2. (3 p) We apply a spin independent external potential with a particular multipole symmetry, i.e., $\nu_l Y_{lm}(\hat{p})$ on a Fermi liquid. That is, the bare quasi-particle energy changes: $\delta\epsilon^{(0)}_{\vec{p},\sigma} = \nu_l Y_{lm}(\hat{p})$. Show carefully that the change of the (full) quasi-particle energy is given by $\delta\epsilon_{\vec{p},\sigma} = \frac{\nu_l}{1+F^s_l} Y_{lm}(\hat{p})$.

3. We consider a Fermi-liquid with two type of contact interactions, $V_1(x-x') = \lambda_1 \delta^{(3)}(x-x')$ and $V_2(x-x') = -\lambda_2 \nabla^2 \delta^{(3)}(x-x')$.

   a) (3 p) Calculate, to leading order, the Landau parameters for both these interactions.

   b) (1 p) For which parameter regime (λ_1, λ_2) is this Fermi liquid unstable?

4. (3 p) From the adiabatic principle, we know that the ground state and one-quasi-particle states of an interacting Fermi liquid are given by

$$|\phi\rangle = U|\Psi_0\rangle \qquad\qquad |\widetilde{\vec{k}\sigma}\rangle = U|\vec{k}\sigma\rangle = U c^\dagger_{\vec{k}\sigma}|\Psi_0\rangle \ ,$$

   where $|\Psi_0\rangle$ is the filled Fermi sea of the non-interacting system, U is the (time-ordered) evolution operator describing the interaction, $\vec{k}$ is a momentum close to the Fermi surface, and $c^\dagger_{\vec{k}\sigma}$ creates a particle in the non-interacting fluid. The operator that creates a quasi-particle can formally be written as $a^\dagger_{\vec{k}\sigma} = U c^\dagger_{\vec{k}\sigma} U^\dagger$. In a Fermi liquid, the quasi-particle states have a *finite* overlap with the states that is formed by creating a *bare* particle, that is

$$Z_k = |\langle \widetilde{\vec{k}\sigma}| c^\dagger_{\vec{k}\sigma}|\phi\rangle|^2$$

   This means that the Green's function takes the form

$$G(\vec{k},\omega) = \frac{Z_k}{\omega - \epsilon_{\vec{k}} + i\delta \mathrm{sgn}(k-k_F)} + \mathrm{reg.}$$

   Show by using this Green's function, that the momentum distribution of the physical electrons n^e_k has a finite jump at the Fermi surface, $n^e_{k_F-\delta k} - n^e_{k_F+\delta k} = Z_k$, in the limit of small δk.