To lowest order in gradients, this gives: ($e=1$)

$$S[\vartheta, A] = \int d\tau\, d^d r \; c_1 [\partial_\tau - \phi]^2 + c_2 [\vec{\nabla}\vartheta + \vec{A}]$$

See AS for der. of constants: $c_1 = V$; $c_2 = \frac{n_s}{2m}$.

n_s is the superfluid density.

We are interested in response to magnetic field,

no el. field: $\phi = 0$, $\partial_\tau A = 0$.

For T high enough we can ignore temp. fluctuations in ϑ: $\partial_\tau \vartheta = 0$.
(quantum)

The maxwell term: $\frac{1}{4}\int d^d r\, d\tau\, F_{\mu\nu}F^{\mu\nu} = \frac{\beta}{2}\int d^d r\, \underset{(\vec{\nabla}\times\vec{A})^2}{\underset{\shortparallel}{B^2}}$

This gives:

$$S[\vartheta, A] = \frac{\beta}{2}\int d^d r \left[\frac{n_s}{m}\left(\vec{\nabla}\vartheta - A\right)^2 + \left(\vec{\nabla}\times\vec{A}\right)^2\right]$$

See AS for integration over ϑ (quadratic):

$$S[A] = \frac{\beta}{2}\sum_q \left(\frac{n_s}{m} + q^2\right) A_q^\perp \cdot A_q^\perp$$

$\hookrightarrow \perp$ to q.

So, the goldstone mode (phase) was integrated out, giving a mass to the A field: $\frac{n_s}{m}$.

We say that a SC is in a <u>Higgs</u> phase

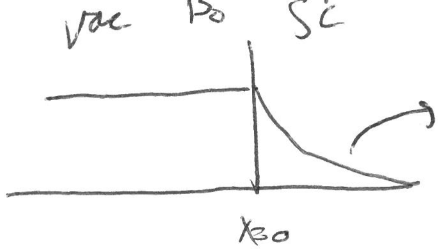
To see the consequences: vary the action:

$$\left(\frac{n_s}{m} + q^2\right) A(q) = 0 \rightsquigarrow \left(\frac{n_s}{m} - \nabla^2\right) A(z) = 0$$

Take curl: $\left(\frac{n_s}{m} - \nabla^2\right) B(z) = 0$ $\quad$ 1st London eq.

When $n_s \neq 0$, there can not be a non-zero ~~con~~ constant $\vec{B}$-field: Meissner effect.

At the edge of a SC in a field:



$B(x) \sim B_0 \exp(-x/\lambda)$

$\lambda = \sqrt{m/n_s}$: penetration depth.

The field diminishes due to screening currents:

$$j(z) = \frac{\delta S}{\delta A} = \frac{\delta}{\delta A(z)} \int d^d z \, \frac{n_s}{2m} A \cdot A = \frac{n_s}{m} A(z)$$

2nd London eq.

$A(z)$ & $B(z)$ have same profile, so currents decrease in the SC; they cancel the $\vec{B}$-field

Zero resistance: $\phi = 0$, so $\vec{E} = -i\partial_\tau \vec{A}$, τ

2nd London eq: $-i\partial_\tau j = \frac{n_s}{m}(-i)\partial_\tau A = \frac{n_s}{m} E$, or in

real time: $\partial_t j = \frac{n_s}{m} E$. j increases in an E-field

Ballistic transport (or: can not have gradients in E)

Bose-Einstein condensation

Concept can be understood in terms of a *free* theory:

$$Z_0 = \int D(\bar{\psi}, \psi)\, e^{-\sum_{a,n} \bar{\psi}_{an}(-i\omega_n + \varepsilon_a - \mu)\psi_{an}},$$

boson field!

Where we assume that the ground state $\varepsilon_0 = 0$, and

$\varepsilon_a > 0$.

For the free theory, we need $\mu < \varepsilon_0 = 0$, otherwise the integral diverges.

The number of particles: $N(\mu) = -\partial_\mu F = T\partial_\mu \log Z$

$$= -\partial_\mu(-T\log Z) = T\partial_\mu \log Z$$

$$* \quad N(\mu) = T\sum_{a,n} \frac{1}{i\omega_n - \varepsilon_a + \mu} = \sum_a n_B(\varepsilon_a), \text{ where}$$

$$n_B(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} - 1} \quad \text{Bose Einstein distribution}$$

For given number of particles, this gives a ~~relation~~ μ as a function of T.

As T goes down, $\beta = \frac{1}{T}$ goes up, so $n_B(\varepsilon)$ goes down; so μ has to increase to compensate.

At some $T = T_c$, even $\mu = 0$ will not suffice to ~~to~~ keep $n_B(\varepsilon \neq \varepsilon_0)$ large enough,

$$\sum_a n_B(\varepsilon_a)\Big|_{\mu=0} \equiv N_1 < N$$

This ~~mean~~ means that one needs $N_0 = N - N_1$ ~~par~~, a macroscopic number of particles in the ground state!

This is Bose-Einstein condensation! Observed in 1995, for Rubidium atoms at $\sim 20\,nK$

To describe this, we have to look at $\psi_0(\tau)$.

However, $\varepsilon_0 = \mu = 0$; and $\omega_0 = 0$, so the integral over $d(\bar\psi_{00}, \psi_{00})$ is unbounded.

~~So, to~~ But, we can ~~use~~ treat ψ_0 as a 'Lagrange multiplier', which sets the total number of particles:

$$S_0[\bar\psi, \psi] = -\bar\psi_0 \beta \mu \psi_0 + \sum_{a \neq 0, n} \bar\psi_{an}(-i\omega_n + \varepsilon_a - \mu)\psi_{an}$$

To get the number of particles, we differentiate wrt μ, and set $\mu = 0$ afterwards:

$$N = -\partial_\mu F\Big|_{\mu=0} = \bar\psi_0 \psi_0 + T \sum_{a \neq 0, n} \frac{1}{i\omega_n - \varepsilon_a} = \bar\psi_0 \psi_0 + N_1$$

So, $\bar\psi_0 \psi_0 = N_0$ is the number of particles in the condensate.