Tutorial on PDEs

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1 Heat equation

1) Derive the heat equation.

2) Solve it in 1D for the following boundary and initial conditions:

$$\begin{cases} \forall t > 0, \quad T(x=0,t) = \pi \text{ and } T(x=L,t) = 0 \\ \forall x \in [0, L], \quad T(x, t=0) = 0 \end{cases}$$

Why can't we directly apply the method of separation of variables ? It may be useful to look for a solution of the form $T(x,t) = u(x) + v(x,t)$ such that :

$$\begin{cases} u(0) = \pi \text{ and } u(L) = 0 \\ \forall t > 0, \quad v(x=0,t) = 0 \text{ and } v(x=L,t) = 0 \\ \forall x \in [0, L], \quad v(x, t=0) = -u(x) \end{cases}$$